Digital Edition of Euclid’s Elements

Written twenty-three centuries ago in Alexandria, Euclid’s Elements stands as the founding document of mathematics. Summing up the singular, revolutionary accomplishments of Greek mathematics of the period 500 to 300 BC, this book established the tradition of distilling mathematical knowledge into the organized form of axiom, definition, and theorem, and with it, the precedent that the test of mathematical truth is the communication and subsequent communal acceptance of a proof. This was clearly a brilliant idea: in the intervening millennia, not one of the theorems in the Elements has been found false, even though standards of rigor have changed.

One wonders why Greek society was so fertile a field for such purely intellectual pursuits. Neither a theorem demonstrating the infinitude of the primes nor the construction of the icosahedron could have had much utility in contemporary economic or political life, yet these were the outcome of central research preoccupations of the Greek mathematical establishment of the times. Whatever the reasons, mathematicians, scientists, and students throughout the ages are the intellectual beneficiaries of this magnificent, prescient work.

The Elements was the standard, if not the sole, textbook for instruction in mathematics from Euclid’s day until sometime in the twentieth century. From it, generations of students learned the deep and beautiful properties of lines, triangles, and circles. But its influence extended far beyond the properties of geometric figures. Through page after page of tightly structured, intricately woven logical argument, the Elements taught the art of reasoning and also the art of solving problems. Perhaps the highest accolade to its influence was that paid by Isaac Newton, who, in founding modern physics, took the Elements as his stylistic and intellectual model. A look at Newton’s *Principia* makes this clear. As in geometry, one begins with axioms, e.g., *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon*. From these one deduces first one and then another theorem, building, brick by brick, layer by layer, an edifice of sure and organized knowledge.
The Bodleian Library, CMI, and Octavo have worked together to produce a digital edition of the oldest surviving manuscript of Euclid’s Elements.

A close reading of the two pages reproduced from the manuscript yields many rewards. In this fine Byzantine calligraphy you see part of Book III, stating theorems four through seven on basic properties of circles. The first theorem reads: 

If in a circle two straight lines which do not pass through the center cut one another, then they do not bisect one another. Notice the diagrams almost identical to the ones that we studied in high school geometry class. Notice also the marginal notes, written in the same hand as the manuscript itself. Like the Elements, they are of ancient origin, though somewhat younger: it was the duty of the scribe to make a fully faithful copy, scribbling included, just as this job is performed today by the camera. The marginal notes (scholia) give us insight into the uses and readership of the Elements in the early days of mathematics.

You can try your hand at identifying the theorems. Knowing that the Greek numbers for 1 through 9 were α, β, γ, δ, ε, ζ, η, θ, ι, with κ for 20, λ for 30, etc., one sees that the beginning of theorem four is signaled by the Δ in the margin. The end is signaled by the phrase ὅπερ ἔδει δεῖξαι — Greek for Q.E.D. For more insight into the manuscript, see Heiberg’s edition of the Elements in Greek. It is available online at www.perseus.tufts.edu. The standard English translation, with excellent commentary, is by Sir Thomas Heath. It is available both online at Perseus and from Dover Publications.

The oldest extant manuscript of the Elements dates to September, 888 AD, when the last ink stroke was penned by Stephen the Clerk, working in Constantinople. We know that it was bought for fourteen gold coins by Arethas of Patrae (later bishop of Caesarea in Capadocia). Its whereabouts are obscure from that time until the seventeenth century, when it was acquired by the Dutch classicist J.P. D’Orville. Part of D’Orville’s library was bought by the Bodleian Library in Oxford in 1804, where the Elements has remained ever since. What you see above are two pages from this manuscript, reproduced from the digital edition prepared by Octavo, a company that specializes in the production of fine digital editions of rare books and manuscripts.
Euclid’s Elements was first printed in 1482 at the Venetian press of Erhard Ratdolf. It went through dozens of editions during the Renaissance. In 1570 the first English translation was made by Sir Henry Billingsley, who in 1596 became Lord Mayor of London. The manuscript of the Bodleian Euclid has been directly accessible to only a few scholars. Now, for the first time, it is easily available in Octavo’s digital edition. The digitization project grew out of conversations between Jim Carlson, President of CMI, Chet Grycz, publisher and CEO of Octavo, and Richard Ovenden, Keeper of the Western Collection at the Bodleian Library. Throughout, this work has been a labor of love, and all of us are pleased that it could be brought to fruition so quickly: a year’s time from concept to publication, with Octavo’s photography taking place at the Bodleian Library from September 20 to October 22 of 2004, under the direction of Hans Hansen, Imaging Director of Octavo and Stefan Grycz, Studio Manager of Octavo UK. The Bodleian Library, the Clay Mathematics Institute, and Octavo are proud to be able to help write one more chapter in the history of publishing the Elements.

The manuscript can be viewed and studied at the Bodleian Library, in digital form at CMI’s offices in Cambridge, Massachusetts, and on the Octavo.com website, where the research facsimile edition, distributed on four CDs, is available. It contains full-resolution images (576 ppi). At a future date, a reading edition with lower resolution images, but with commentary and translation, will be released at a price below $100. This edition will include the authoritative Heiberg edition of the Elements and the Heath translation, all keyed by page to the manuscript. Finally, a 100% scale edition of the manuscript will later be made freely available. CMI and Octavo plan to prepare digital editions of selected mathematical manuscripts of historical importance. It is our hope that publishing such works will renew interest in the historical roots of mathematics, will revivify its teaching, and will increase its wider appreciation.

Euclid and his Heritage, a conference organized by CMI, the Bodleian Library, and Octavo will be held October 7–8, 2005 at St. Catherine’s College in Oxford. The conference will present the digital edition and will bring together historians, scholars, mathematicians, teachers, and other interested individuals for an examination of the history and influence of the Elements. All are invited.

Attendees will receive a facsimile of selected parts of the manuscript, which will play an important role in the program. To open the conference, noted Oxford paleographer Nigel Wilson will discuss in detail two pages of the facsimile. With him, we will see what can be learned from a close reading of an original manuscript. For more information, including hotel accommodations and registration, see www.claymath.org/euclid.