Mean Curvature Flow
September 26 – 30, 2016

Abstracts of Talks

Sigurd Angenent (University of Wisconsin)

Title: Mean curvature flow from cones

Abstract: A forward evolution by MCF of a smooth cone in R^n is sometimes given by smooth self similar expanding solutions of MCF. In some cases there are also non self similar solutions. I will show how to construct examples these non self similar solutions, and show examples where these non self similar solutions appear in large dimensional families.

Jacob Bernstein (John Hopkins University)

Title: Surfaces of low entropy

Abstract: The entropy of a surface is a geometric measure of its complexity introduced by Colding and Minicozzi. The entropy is non-increasing along a mean curvature flow and is closely related to singularity formation of the flow. We discuss some recent results showing that closed surfaces with low entropy are simple in various senses. This is joint work with L. Wang.

Yoshikazu Giga (University of Tokyo)

Title: A level-set crystalline mean curvature flow of hypersurfaces

Abstract: An anisotropic mean curvature flow is important in materials science. If the interfacial energy is singular, the speed becomes a nonlocal quantity. For evolution of a curve it has been well-studied. Especially, a level-set flow, which is a convenient notion to track the evolution after it develops singularities, was constructed around 15 years ago by Mi-Ho Giga and myself. However, it is quite recent that an evolution of a surface by such an singular anisotropic interfacial energy is studied in a level-set framework, even for crystalline energy. In this talk we give a way to construct a level-set flow for such a problem. This is a joint work with N. Pozar (Kanazawa).

Bruce Kleiner (Courant Institute, NYU)

Title: Uniqueness of weak solutions to Ricci flow, and Perelman's convergence conjecture

Abstract: In his proof of Thurston's geometrization conjecture, Perelman proved the existence of a Ricci flow with surgery starting from any given compact smooth Riemannian 3-manifold. In the same papers, he conjectured that when the surgery parameters are sent to zero, the flow with surgery converges to a limiting "flow through singularities", yielding a canonical generalized Ricci flow. I will discuss the proof of Perelman's convergence conjecture, and comment on related uniqueness questions for weak solutions to Ricci flow and mean curvature flow. This is joint work with Richard Bamler.

Robert B. Kohn (Courant Institute, NYU)

Title: Prediction without probability: a PDE approach to some two-player games from machine learning

Abstract: In the machine learning literature, one approach to "prediction" assumes that advice is available from a finite number of "experts." The best prediction in this setting is the one that "minimizes regret", i.e. minimizes the worst-case shortfall relative to the best performing expert. My talk discusses a particular problem of this type, which takes the form of a randomized-strategy two-player game. I'll explain recent work with Nadejda Drenska, which addresses this problem using ideas from optimal control and partial differential equations. The main idea is to consider a suitable continuum limit, and to characterize the value

function using a nonlinear PDE. While this machine learning problem has nothing to do with motion by curvature, our overall approach is similar to my 2006 work with Serfaty concerning a deterministic-control-based approach to motion
by curvature.

Felix Otto (MPIM Leipzig)

Title: Convergence of the thresholding scheme for multi-phase mean curvature flow.

Abstract: We consider the thresholding scheme, a time discretization for mean curvature flow introduced by Bence-Merriman-Osher; and prove a convergence result in the multi-phase case. The result establishes convergence towards a weak formulation in the framework of sets of finite perimeter. Multi-phase mean-curvature flow is a model for grain growth, especially when one allows for surface tensions that depend on the pairs of grains, as we do.

The proof is based on the interpretation of the thresholding scheme as a minimizing movement scheme, which means that the thresholding scheme preserves the structure of (multi-phase) mean curvature flow as a gradient flow w.\ r.\ t.\ the total interfacial energy. More precisely, the thresholding scheme is a minimizing movement scheme for an energy functional that Γ-converges to the total interfacial energy (joint work with Selim Esedoglu).

Our proof is similar in spirit to the convergence results of Almgren-Taylor-Wang and Luckhaus Sturzenhecker of another minimizing movement scheme for mean curvature flow. In particular, ours is a conditional convergence result, in the sense that we assume that the energy of the approximation converges to the energy of the limit. In addition, we appeal to an argument of De Giorgi to show that the limit satisfies the gradient flow structure in the sense of Brakke.

This is joint work with Tim Laux.

Felix Schulze (University College London)

Title: Ricci flow from spaces with isolated conical singularities

Abstract: Let (M,g_0) be a compact n-dimensional Riemannian manifold with a finite number of singular points, where at each singular point the metric is asymptotic to a cone over a compact (n-1)-dimensional manifold with curvature operator greater or equal to one. We show that there exists a smooth Ricci flow starting from such a metric with curvature decaying like C/t. The initial metric is attained in Gromov-Hausdorff distance and smoothly away from the singular points. To construct this solution, we desingularize the initial metric by glueing in expanding solitons with positive curvature operator, each asymptotic to the cone at the singular point, at a small scale s. Localizing a recent stability result of Deruelle-Lamm for such expanding solutions, we show that there exists a solution from the desingularized initial metric for a uniform time T>0, independent of the glueing scale s. The solution is then obtained by letting s->0. We also show that the so obtained limiting solution has the corresponding expanding soliton as a forward tangent flow at each initial singular point. This is joint work with P. Gianniotis.

Peter Topping (University of Warwick)

Title: A new pseudolocality theorem for Ricci flow

Abstract: Perhaps the most remarkable result in Ricci flow is Perelman's pseudolocality theorem. We will give an overview of this result, and announce a new pseudolocality theorem. Joint work with Miles Simon.

Lu Wang (University of Wisconsin)

Title: Asymptotic structure of self-shrinkers

Abstract: We show that each end of a noncompact self-shrinker in Euclidean 3-space of finite genus is

smoothly asymptotic to either a regular cone or a self-shrinking round cylinder.

Mu-Tao Wang (Columbia University)

Title: On the stability of higher codimensional mean curvature flows

Abstract: Simon's general asymptotic theorem for gradient flows implies a stability result for the mean curvature flow near a compact stable minimal submanifold, under the C^2 norm. In view of the special feature of submanifold geometry, it is desirable to have a stability theorem that requires lower regularity. In joint work with Chung-Jun Tsai, we investigate an optimal stability theorem under the Lipschitz norm. In particular, a result for calibrated submanifolds and its generalization for general stable minimal submanifolds will be discussed in this talk.