

Algebraic Geometry: Old and New
September 26 – 30, 2016

Abstracts of Talks

Christian Böhning (University of Warwick)

Title: Stable non-rationality for some conic bundles over rational surfaces and rational threefolds

Abstract: Using Brauer type obstructions and the degeneration method due to Voisin, Colliot-Thelene, Pirutka et al., we discuss rationality properties of certain conic bundles over P^2 and P^3 . This is joint work (in progress) with Hans-Christian von Bothmer, Asher Auel and Alena Pirutka.

Tom Coates (Imperial College London)

Title: Mirror symmetry and Fano manifolds

Abstract: I will outline a program -- joint work with Corti, Galkin, Golyshev, Kasprzyk, and others -- to find and classify Fano manifolds using mirror symmetry. I will describe recent progress in this direction, including in dimension 4. The latter is work by and joint work with Prince, Kasprzyk, and Kalashnikov.

Kento Fujita (Kyoto University)

Title: A valuative criterion for uniform K-stability of log Fano pairs

Abstract: It's a difficult problem whether a given Fano manifold is K-(semi)stable or not. In this talk, I will introduce "valuative criteria", of those stability conditions, which seem relatively easy.

Mark Gross (University of Cambridge) and **Bernd Siebert** (Universität Hamburg)

Title: An intrinsic mirror symmetry construction.

Abstract: We will survey recent progress in our joint program for understanding mirror symmetry, leading to a general mirror symmetry construction for maximal log Calabi-Yau pairs and maximally unipotent degenerations of Calabi-Yau varieties. We do this by introducing a variant of log Gromov-Witten invariants called "punctured invariants" developed in joint work with Abramovich, Chen. We will then explain how to use these invariants to give a general construction of mirrors by building the coordinate ring of the mirror explicitly in terms of these invariants.

Dominic Joyce (University of Oxford)

Title: New Donaldson-Thomas style counting invariants for Calabi-Yau 4-folds

Abstract: Pantev, Toen, Vezzosi and Vaquie arXiv:1111.3209 introduced the notion of "k-shifted symplectic derived schemes and stacks" in Derived Algebraic Geometry. They showed that moduli stacks of coherent sheaves and complexes on a Calabi-Yau m-fold Y are $(2-m)$ -shifted symplectic. So in particular Calabi-Yau 3-fold moduli stacks are -1-shifted symplectic, and Calabi-Yau 4-fold moduli stacks are -2-shifted symplectic. In previous work with Ben-Bassat, Brav, Bussi, Dupont, Meinhardt, and Szendroi we studied -1-shifted (3-Calabi-Yau) geometry and generalizations of

Donaldson-Thomas theory. Today we move on to the -2 -shifted case.

Using a "shifted symplectic Darboux Theorem" by Brav, Bussi and Joyce, we prove that a -2 -shifted symplectic derived scheme X over \mathbb{C} can be given the structure of a "derived smooth manifold" (d-manifold, or M-Kuranishi space) X^* , uniquely up to bordisms of X^* fixing the underlying topological space.

If X is proper and has an "orientation" (similar to Kontsevich-Soibelman orientation data in the 3-Calabi-Yau case), then X^* is a compact, oriented derived manifold, and so has a virtual class (e.g. in bordism), which is an integer if $\text{vdim } X = 0$.

This should give virtual classes for proper Calabi-Yau 4-fold moduli schemes, and lead to new Donaldson-Thomas style invariants "counting" (semi)stable coherent sheaves on a Calabi-Yau 4-fold Y , which will be unchanged under continuous deformations of Y .

This is joint work with Dennis Borisov. It is related to work of Cao and Leung in arXiv:1407.7659.

Anne-Sophie Kaloghiros (Brunel University)

Title: The Sarkisov program for Mori fibred Calabi-Yau pairs

Abstract: A Calabi-Yau (CY) pair (X, D) is a pair of a normal variety X and a reduced divisor D such that $K+D$ is a Cartier divisor linearly equivalent to 0; two such pairs will be considered equivalent when there is a volume preserving birational map between them. If X itself has a structure of Mori fibre space-- that is if X is a 'good' family of Fano varieties— then (X, D) is a Mori fibred CY pair. Such a pair is the end product of two distinct Minimal Model Programs: on the one hand, it is a $K+D$ -minimal model, and on the other it is the end product of a classical MMP. In this talk, I will present results on volume preserving maps between CY pairs, and in particular, a Sarkisov-type factorisation theorem for maps between Mori fibered CY pairs.

Conan Leung (Chinese University of Hong Kong)

Title: Categorical Plucker formula and homological projective dual

Abstract: A generalised Plucker formula describes changes of intersection numbers of complex Lagrangian under Mukai flop. In a recent joint work with Jiang and Xie, we generalise this to the level of derived category of coherent sheaves.

Yuchen Liu (Princeton University)

Title: The volume of Kähler-Einstein \mathbb{Q} -Fano varieties

Abstract: A complex projective variety is \mathbb{Q} -Fano if it has klt singularities and the anti-canonical divisor is \mathbb{Q} -Cartier and ample. Starting from dimension 2, the anti-canonical volume of a \mathbb{Q} -Fano variety can be arbitrarily large, e.g. weighted projective spaces. Recently, Fujita showed that an n -dimensional Kähler-Einstein \mathbb{Q} -Fano variety has volume at most $(n+1)^n$. In this talk, I will discuss a refinement of Fujita's volume upper bounds involving invariants of the local singularities. If time permits, I will also talk about an equivalent relation between K -semistability and de Fernex-Ein-Mustață type inequalities. Part of this work is joint with Chi Li.

Eduard Looijenga (Utrecht University, Tsinghua University)

Title: Basic Tate extensions in the cohomology of Baily-Borel compactifications

Abstract: An automorphic vector bundle on a Shimura variety will in general not extend to as a bundle to its Baily-Borel compactification. Yet Goresky and Pardon showed that the Chern classes of such a bundle do so in a natural manner and asked whether these classes lie in rational cohomology. We show that these extensions have a natural description in the setting of mixed Hodge theory and may not lie in rational cohomology. We apply this to the primitive part of the stable cohomology of the Baily-Borel compactification of the ppav's to show that they yield all the basic Tate extensions.

Emanuele Macri (Northeastern University)

Title: Tilting of bounded t-structures and applications

Abstract: We will review the construction of tilting for bounded t-structures in the derived category of coherent sheaves on a smooth projective variety. We will discuss a few applications of this: Bayer's proof of the Brill-Noether Theorem, the description of nef cones of Hilbert schemes of points on surfaces, and the proof for a conjecture of Huybrechts on derived categories of cubic fourfolds. The talk is based on joint work with Bayer and on work in progress with Bayer, Lahoz, and Stellari.

Yuji Odaka (Kyoto University)

Title: On compactifying the moduli spaces of Kahler-Einstein varieties

Abstract: Around 2009-2010, the speaker found that “Kollár-Shepherd-Barron-Alexeev” type project moduli of ample canonical class varieties, can be regarded as a special case of moduli of K-stable/Kähler-Einstein (KE) varieties, through birational geometric study of K-stability.

Later we, with C.Spotti and S.Sun, constructed “similar” (conjecturally projective) canonically compactified moduli spaces for Fano manifolds (cf., C. Spotti and X. Wang’s talk) in 2-dim case and generalized later. This uses the strong results in the Kahler-Einstein geometry (cf., S.Sun’s talk).

After some possible brief review of such earlier works, I will probably focus on “tropical geometric compactifications” of, again, moduli spaces of KE varieties (arXiv:1406.7772) - and its recent ongoing developments (in preparation). The compactifications attach “tropical moduli spaces” as boundaries to e.g. M_g, A_g . This is also closely related to the geometric “Strominger-Yau-Zaslow” mirror symmetry, the “Gross-Siebert program”.

Bernd Siebert (Universität Hamburg) and **Mark Gross** (University of Cambridge)

Title: An intrinsic mirror symmetry construction.

Abstract: We will survey recent progress in our joint program for understanding mirror symmetry, leading to a general mirror symmetry construction for maximal log Calabi-Yau pairs and maximally unipotent degenerations of Calabi-Yau varieties. We do this by introducing a variant of log Gromov-Witten invariants called “punctured invariants” developed in joint work with Abramovich, Chen. We will then explain how to use these invariants to give a general construction of mirrors by building the coordinate ring of the mirror explicitly in terms of these invariants.

Cristiano Spotti (Aarhus University)

Title: Kähler–Einstein metrics on smoothable \mathbb{Q} -Fano varieties

Abstract: In this talk I will discuss the problem of the existence of Kähler–Einstein metrics on \mathbb{Q} -Gorenstein smoothable Fano varieties (joint work with S. Sun and C. Yao) and I will explain their relevance for the construction of compact moduli spaces of K-stable Fanos, focusing on the two dimensional case (joint with Y. Odaka and S. Sun). Finally, I will show some possible applications to build "cscK generalised conifold transitions" (joint with C. Arezzo).

Jacopo Stoppa (SISSA Trieste)

Title: K-stability, equivariance and filtrations

Abstract: I will discuss the general equivariance problem in the theory of K-stability for projective varieties, its applications, and its connections to alternative notions of stability based on filtrations of the coordinate ring.

Song Sun (University of Stony Brook)

Title: Geometric flows and algebraic stability

Abstract: We study the asymptotic behavior of certain canonical geometric flows on projective manifolds, and relate it to algebraic stability and “optimal degenerations”. This leads to a new proof of the Kähler-Einstein result on Fano manifolds via Kähler-Ricci flow. We will also explain potential applications in the existence problem of extremal Kähler metrics on general polarized manifolds via the Calabi flow. Some interesting questions will be discussed. Joint work with X. Chen and B. Wang.

Alessandro Verra (Università Roma Tre)

Title: The universal K3 surface of genus 14 via cubic fourfolds.

Abstract: In complex projective geometry the interplay of K3 surfaces and cubic fourfolds is a well known topic, possibly related to the rationality problem for a cubic fourfold. This was recently attracting the attention of many authors. In the talk a survey on the more recent results is given and a special case of interest is considered. This is the family of (conjecturally rational) cubic fourfolds X containing a 3-nodal scroll R of degree 7. Such a family is associated to the moduli space F_{14} of K3 surfaces polarized in genus 14. The new result presented in the talk is that the moduli space of pairs (X,R) is rational and birational to the universal K3 surface over F_{14} . (Joint work with G. Farkas).

Xiaowei Wang (Rutgers University)

Title: Moduli space of Fano Kähler-Einstein manifolds

Abstract: In this talk, we will discuss our construction of compact Hausdorff Moishezon moduli spaces parametrizing smoothable K-stable Fano varieties. The solution relies on the recent solution of the Yau-Tian-Donaldson conjecture by Chen-Donaldson-Sun and Tian. In particular, we prove the uniqueness of the degeneration of Fano Kähler-Einstein manifolds and more algebraic properties that are needed to construct a good algebraic moduli space. (This is a joint work with Chi Li and Chenyang Xu).

Jaroslav Wiśniewski (University of Warsaw)

Title: Flag varieties, a geometric characterization and rigidity

Abstract: Flag varieties can be characterized as these Fano manifolds whose all extremal contractions are smooth P^1 fibrations. Subsequently they are (globally) rigid in families of Fano manifolds. I will report on these results obtained in collaboration with Gianluca Occhetta, Luis E. Solá Conde, Kiwamu Watanabe, and Andrzej Weber.