

**Algebraic Topology: Manifolds Unlocking Higher Structures**  
**28 September – 02 October, 2015**

**Abstracts of Talks**

**Clark Barwick** (Massachusetts Institute of Technology)

*Title:* Modes of equivariance

*Abstract:* The Mackey functor model of equivariant stable homotopy makes it possible to contemplate a wide array of other modes of equivariance – replete with all the multiplicative trimmings (whose import was of course emphasized by Hill, Hopkins, and Ravenel). Steered by my favorite examples, I'll survey the results of the forthcoming book with Dotto, Glasman, Nardin, and Shah, as well as a few satellite papers.

**Julie Bergner** (University of California, Riverside)

*Title:* Complete Segal objects and higher homotopical categories

*Abstract:* The notion of (infinity,  $n$ )-category, which should be thought of as a higher category up to homotopy, can be realized through a number of different precise models. One of the expected features of a good model is that (infinity,  $n$ )-categories should be equivalent to categories enriched in (infinity,  $n-1$ )-categories. In recent work with Rezk, we establish this comparison for the  $\Theta_n$ -space model. In establishing this equivalence, we develop new models which generalize the idea of Segal categories and complete Segal spaces as weakly enriched categories.

**Andrew Blumberg** (University of Texas, Austin)

*Title:* Arithmetic duality for algebraic K-theory and the algebraic K-theory of the sphere spectrum

*Abstract:* I will describe joint work with Mike Mandell that establishes a spectral version of Tate-Poitou duality for the algebraic K-theory spectra of number rings. I will explain how this result can be applied to study the homotopy type of the algebraic K-theory of the sphere spectrum (i.e., Waldhausen's A-theory of a point).

**John Francis** (Northwestern University)

*Title:* A proof of the cobordism hypothesis

*Abstract:* The cobordism hypothesis -- after Baez-Dolan, Costello, Hopkins-Lurie, and Lurie -- classifies extended topological quantum field theories in terms of their values on a point. I'll describe a proof, currently in preparation, which follows from the theory of factorization homology of (infinity,  $n$ )-categories. This is joint work with David Ayala.

**Dan Freed** (University of Texas, Austin)

*Title:* Reflection Positivity in Extended Field Theory

*Abstract:* The classification of invertible topological phases in condensed matter physics led Mike Hopkins and me to an ongoing mathematical investigation of reflection positivity, the Wick-rotated

notion of unitarity in quantum field theory. I will report on our progress and also on the application which motivated the project.

**Soren Galatius** (Stanford University)

*Title:* Homological stability and non-stability for moduli spaces of closed manifolds

*Abstract:* Two closed manifold  $M$  and  $M'$  of dimension  $2n$  are said to be stably diffeomorphic if they become diffeomorphic after replacing each with its connected sum with a finite number of copies of  $S^n \times S^n$ . In general there doesn't seem to be any good maps between the classifying spaces  $B\text{Diff}(M)$  and  $B\text{Diff}(M')$ , but nevertheless it turns out that they have isomorphic rational cohomology in a range of degrees, provided the numbers  $(-1)^n \chi(M)$  and  $(-1)^n \chi(M')$  are both sufficiently large,  $n > 2$  and the manifolds are simply connected. The analogous statement in integral cohomology is false, but with  $p$ -local coefficients it is true provided in addition that the  $p$ -adic valuations of  $\chi(M)$  and  $\chi(M')$  agree. This is joint work with Oscar Randal-Williams.

**Kathryn Hess** (EPFL)

*Title:* Waldhausen K-theory and topological coHochschild homology

*Abstract:* I will present joint work with Brooke Shipley, in which we have defined a model category structure on the category of comodule spectra for the suspension spectrum of  $X$  such that the K-theory of the associated Waldhausen category of homotopically finite objects is naturally weakly equivalent to the usual Waldhausen K-theory of  $X$ ,  $A(X)$ . I will describe the relation of this comodule approach to  $A(X)$  to the more familiar description in terms of module spectra for the free infinite loop space on  $X$ . I will also explain the construction and properties of the topological coHochschild homology of  $X$ , which is a potentially interesting approximation to  $A(X)$ .

**Lars Hesselholt** (Nagoya University and University of Copenhagen)

*Title:* Topological Hochschild homology and periodicity

*Abstract:* I will explain that topological Hochschild homology enjoys a periodicity more basic than Bott periodicity and show that this leads to a natural home for Deninger's formula for the Hasse-Weil zeta function of a scheme smooth and proper over a finite field as a regularized determinant.

**Nick Kuhn** (University of Virginia)

*Title:* Hurewicz maps for infinite loop spaces: theorems, examples, and conjectures

*Abstract:* In a 1958 paper, Milnor observed that then new work by Bott allowed him to determine which spheres admit a vector bundle with non-trivial top Stiefel-Whitney class. This can be interpreted as a calculation of the mod 2 Hurewicz map for the classifying space  $BO$ . The infinite loop space  $BO$  is the 0th space of a spectrum  $bo$ , so this Hurewicz map fits the form:

$$h: \pi_*(b) \rightarrow H_*(B; \mathbb{Z}/p),$$

where  $B$  is the 0th space of a spectrum  $b$ .

I have been studying such Hurewicz maps for nice generalized homology theories by relating the Adams filtration of the domain to a filtration of the range induced by an 'augmentation ideal' filtration of the commutative  $S$ -algebra associated to  $B$ .

When specialized to mod  $p$  homology, my general results have some tidy consequences:

- If  $H^*(b; \mathbb{Z}/p)$  is a finitely presented module over the Steenrod algebra, then the image of the Hurewicz map in  $H_*(B; \mathbb{Z}/p)$  will be finite dimensional.
- Milnor's theorem follows from a glance at Adams SS charts for  $bo$ .
- The analogous calculation can be similarly done for  $tmf$ .
- One recovers Wilson's theorem that certain infinite loopspaces  $BP\langle n \rangle_k$  are atomic spaces.

The development of the augmentation ideal filtration, with needed properties with respect to homotopy invariance, composition, and base change, is joint work with Luis Pereira.

**Wolfgang Lueck** (Universität Bonn)

*Title:* Aspherical manifolds, what we know and what we do not know.

*Abstract:* Aspherical closed manifolds arise very often in topology, for instance in low dimensional topology, closed Riemannian manifolds with non-positive sectional curvature and so on. We want to give a survey about open problems (and their status) such as the Borel Conjecture about topological rigidity, the Novikov Conjecture about the topological invariance of higher signatures, the Singer Conjecture about the distribution of  $L^2$ -Betti numbers, approximation of  $L^2$ -torsion, and the realizability of Poincaré duality groups as fundamental groups of aspherical closed manifolds. Moreover, we present results about the rational homotopy groups of the group of diffeomorphisms and homeomorphisms of aspherical closed manifolds and the problem which hyperbolic groups have the standard sphere as boundary.

**Ib Madsen** (University of Copenhagen)

*Title:* Automorphisms of manifolds and graph homology.

*Abstract:* I will explain joint work with Alexander Berglund that calculates the rational cohomology of spaces of homotopy automorphisms and block diffeomorphisms for 2d-dimensional "surfaces". I will explain the relation to the work of Galatius and Randal-Williams on the space of diffeomorphisms. The method is based on Quillen's version of rational homotopy theory, and on surgery theory. The end results may be formulated in terms of Kontsevich's graph homology and ultimately in terms of cohomology of certain discrete groups.

**Oscar Randal-Williams** (University of Cambridge)

*Title:* Tautological rings for high-dimensional manifolds

*Abstract:* The cohomology of the classifying space  $B\text{Diff}(M)$  of the group of diffeomorphisms of a manifold  $M$  may be considered as the ring of characteristic classes of smooth fibre bundles with fibre  $M$ . This ring is difficult to understand, but when  $M$  is an orientable surface the close connection between  $B\text{Diff}(M)$  and the moduli space of Riemann surfaces means that a lot is known. In this case, algebraic geometers have found it productive to focus not on all the cohomology but a certain subring, the "tautological ring", containing the geometrically interesting classes. One can make a similar definition for manifolds of higher dimension. I will explain all these terms, and discuss some recent results on the large scale structure of these tautological rings. This is joint work with Ilya Grigoriev and Soren Galatius.

**Marco Schlichting** (University of Warwick)

*Title:* Homology stability of  $SL_n$  and a conjecture of Bass

*Abstract:* We improve homology stability ranges for  $SL_n$  over rings with many units. This allows us to prove a conjecture of Bass from 1973 on stability of unstable Quillen K-theory of such rings.

**Chris Schommer-Pries** (MPIM Bonn)

*Title:* Extended topological field theory in dimension three

*Abstract:* I will describe recent advances in the classification of extended topological field theories which have lead to a nearly complete understanding in dimension three. This includes work in the fully local case based on methods from the Baez-Dolan-Lurie cobordism hypothesis (joint with C. Douglas and N. Snyder), as well as a more direct "generators and relations" approach for partially extended 3D TFTS (assigning values only to 3-manifolds, surfaces, and 1-manifolds, joint with B. Bartlett, C. Douglas, and J. Vicary). If time permits I will describe how these techniques inform our understanding of more exotic structures such as "non-compact" (aka "punctured") and non-semisimple TFTs.

**Peter Teichner** (University of California, Berkeley and MPIM Bonn)

*Title:* Higher order intersection invariants for 4-manifolds

*Abstract:* This will be a survey of recent work with Jim Conant and Rob Schneiderman. We use iterated Whitney disks to define new invariants, only present in 4 dimensions. They extend Wall's self-intersection invariant and provide higher order obstructions for representing 2-dimensional homotopy classes by embeddings. We'll also discuss a relation to quantum invariants via the Kontsevich integral.

**Nathalie Wahl** (University of Copenhagen)

*Title:* Three points of view on string topology

*Abstract:* String topology is concerned with a certain multiplicative structure existing on the chains on the free loop space of a manifold. I'll explain how approaching this structure from three rather different points of view, one algebraic, one geometric and one inspired by physics, all lead to considering the same compactification of the moduli space of Riemann surfaces.