Nonlocal PDEs
September 25 – 29, 2017

Abstracts of Talks

Xavier Cabre (ICREA and UPC, Barcelona)

Title: Nonlocal minimal cones and surfaces with constant nonlocal mean curvature

Abstract: The talk will be concerned with hypersurfaces of R^n with zero, or constant, nonlocal mean curvature. This is the equation associated to critical points of the fractional s-perimeter. We prove that half spaces are the only stable s-minimal cones in R^3 for s sufficiently close to 1. We will then turn to the nonlocal analogue of the Alexandrov result characterizing spheres as the only closed embedded hypersurfaces in R^n with constant mean curvature. Finally, we will describe results establishing the existence of periodic Delaunay-type cylinders in R^n, as well as periodic lattices made of near-spheres, with constant nonlocal mean curvature.

Maria del Mar González (Universidad Autónoma de Madrid)

Title: Non-local ode: an application to the singular fractional Yamabe problem in conformal geometry

Abstract: The singular fractional Yamabe problem poses the question of finding a conformal constant fractional curvature metric with prescribed singularities. From the PDE point of view, this is a semilinear equation with fractional Laplacian and power type right hand side. We will consider the cases when the singular set is a finite number of points and a smooth, closed submanifold. In both problems, a crucial ingredient is the construction of a suitable model solution for a fractional ODE.

Cyril Imbert (ENS Paris)

Title: The weak Harnack inequality of the Boltzmann equation without cut-off

Abstract: In this talk I will present a weak Harnack inequality and H\"older estimates for a large class of kinetic integro-differential equations. We will see how the Boltzmann equation without cut-off can be written in this form and satisfies our assumptions provided that the mass density is bounded away from vacuum and mass, energy and entropy densities are bounded above. As a consequence, we will see how to derive a local H\"older estimate and a quantitative lower bound for solutions of the (inhomogeneous) Boltzmann equation without cut-off.

Tianling Jin (Hong Kong University of Science and Technology)

Title: On the isoperimetric quotient over scalar-flat conformal classes

Abstract: Let *(M,g)* be a smooth compact Riemannian manifold of dimension n with smooth boundary ∂M. Suppose that *(M,g)* admits a scalar-flat conformal metric. We prove that the supremum of the isoperimetric quotient over the scalar-flat conformal class is strictly larger than the best constant of the isoperimetric inequality on Euclidean space, and consequently is achieved, if either (i) $n\ge 12$ and ∂M has a nonumbilic point; or (ii) $n\ge 10$, ∂M is umbilic and the Weyl tensor does not vanish at some boundary point.

Moritz Kassmann (Universität Bielefeld)

Title: Nonlocal energy forms and function spaces

Abstract: We discuss function spaces that are defined with the help of nonlocal quadratic forms. One aim of the talk is to investigate conditions under which such a form is comparable with the quadratic form that defines the seminorm of a fractional Sobolev space. We provide several examples including those with singular measures and one example related to the study of the Boltzmann equation. We outline the proof of comparability in the latter case, which involves chaining and renormalization arguments. We explain the significance of the aforementioned comparability results with regard to Dirichlet forms and regularity of solutions to integrodifferential equations. A second aim of the talk is the presentation of a new nonlocal trace theorem. Finally, we provide a convergence result for the transition from nonlocal to local boundary value problems. The talk is based on joint projects with Bartek Dyda, with Kai-Uwe Bux and Tim Schulze, and with Paul Voigt.

Dennis Kriventsov (New York University)

Title: Regularity in time for fully nonlinear, nonlocal (and local) parabolic equations

Abstract: A significant difference between local and nonlocal parabolic equations is that solutions of the latter are not generally smooth in the time variable: their smoothness is constrained by the regularity of the boundary data. Even for very simple nonlocal parabolic equations, then, typical arguments for establishing regularity in time (such as differentiating the whole equation with respect to time to obtain an equation for the time derivative) do not yield useful results. I will present a method which gives almost optimal time regularity, bypassing these difficulties. This argument requires little structure in the equation: it applies to fully nonlinear (nonconvex) equations; it can be used to prove some new results even in the local setting; and it seems applicable in other contexts, such as parabolic free boundary problems. I will also show some easy corollaries of the result, based on reducing parabolic problems to elliptic ones. This is based on joint work with Hector Chang-Lara.

Jean-Michel Roquejoffre (University of Toulouse)

Title: Front propagation driven by a line of fast diffusion: a property of the level sets.

Abstract. The situation is the following: a line, having a strong diffusion on its own, exchanges mass with a reactive medium, in our case a two-dimensional strip. A front propagates both on the line and in the strip, and one wishes to describe its shape. This setting was proposed (collaboration with H. Berestycki and L. Rossi) as a model of how biological invasions can be enhanced by transportation networks.

Numerical simulations, due to A.-C. Coulon, reveal an a priori surprising property: the solution is not monotone in the direction orthogonal to the strip. The goal of

Ovidiu Savin (Columbia University)

Title: Rigidity results for non-local phase transitions

Abstract: We consider non-local phase transitions and their connection with minimal surfaces, and then discuss about a version of De Giorgi's conjecture for the case of the fractional Laplacian of order $s \in (0,1/2)$.

Joaquim Serra (ETH Zürich)

Title: The De Giorgi conjecture for the half-Laplacian in dimension 4

Abstract: The famous De Giorgi conjecture for the Allen-Cahn equation states that global monotone solutions are 1D if the dimension is less than 9. This conjecture is motivated by a deep connection between Allen-Cahn and minimal surfaces, plus classical results about the structure of global minimal graphs. The analogue of this conjecture in half-spaces can be reduced to study the problem in the whole space for the Allen-Cahn equation with the half-Laplacian. In this talk I will present a recent result with A. Figalli where we prove the validity of the De Giorgi conjecture for stable solutions in dimension 3.As a corollary, we establish the De Giorgi conjecture for the half-Laplacian in dimension 4.

Yannick Sire (Johns Hopkins University)

Title: Asymptotic limits for the fractional Allen-Cahn equation and stationary nonlocal minimal surfaces

Abstract: I will describe an asymptotic result on the fractional Allen-Cahn equation involving the fractional laplacian. The limiting interface is a stationary nonlocal minimal surface. A striking phenomenon is a strong convergence result due to a geometric measure theory result by Marstrand.

Enrico Valdinoci (University of Melbourne)

Title: Crystal dislocation, nonlocal equations and fractional dynamical systems

Abstract: We study heteroclinic and multibump orbits for a system of equations driven by a nonlocal operator. Our motivation comes from the study of the atom dislocation function in a periodic crystal, according to the Peierls-Nabarro model. The evolution of the dislocation function can be studied by analytic techniques of fractional Laplace type. At a macroscopic scale, the dislocations have the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior potential.

Such potential turns out to be either attractive or repulsive, depending on the mutual orientation of the dislocations, and the attractive potentials generate "particle collisions" in finite time. After the collisions, the system relaxes to the equilibrium exponentially fast, and the associated steady states provide a natural setting for the study of dynamics and chaos in a fractional framework.

Alexis Vasseur (University of Texas, Austin)

Title: Regularity theory for non local in time operators

Abstract: In this talk, we will present applications of the De Giorgi method to show the regularity of solutions to nonlocal operators. We will focus on the case of fractional derivatives in time. Those equations are important for the modeling of memory effects as hysteresis. This is a joint work with Mark Allen and Luis Caffarelli.

Juan Luis Vazquez (Universidad Autónoma de Madrid)

Title: Nonlinear fractional diffusion, from older to recent work

Abstract: In this talk I will report on progress made by the author and collaborators on the topic of nonlinear parabolic equations involving long distance interactions in the form of fractional Laplacian operators. Some important questions are adressed: modeling, uniqueness, regularity, FB and finite propagation, asymptotic behavior, bounded domains. Differences with standard diffusion have been specially examined.

Jun-cheng Wei (University of British Columbia)

Title: Counter-examples to De Giorgi Conjecture for Fractional Allen-Cahn

Abstract: We consider the De Giorgi conjecture for fractional Allen-Cahn
$$ (-\Delta)^s u= u- u^3 \ \mbox{in} \ R^N, \ 0<s<1$$

For $s>\frac{1}{2}$ and $N=9$ we construct counterexamples and for $N=8$ we construct nontrivial global minimizers. The key ingredient is a new gluing scheme treating high dimensional concentrations for the fractional elliptic equations. We also discuss how the non-locality adjusts the ends of the catenoids and Lawson's cones. (Joint work with Hardy Chan and Y. Liu.)