LOCAL SMOOTHING IN THE PRESENCE OF LOTS OF TRAPPING

MACIEJ ZWORSKI

Abstract: The Schrödinger propagator, $\exp(-it\Delta)$, on $\mathbf{R}^n$ is unitary on any Sobolev space so regularity is not improved in propagation. Remarkably, and as has been known for about 20 years, the regularity improves when we integrate in time and cut-off in space:

$$\int_0^T \|\chi \exp(-it\Delta)u\|^2_{H^{1/2}} dt \leq C\|u\|^2_{L^2} \, . \quad \chi \in C^\infty_{\rm c}(\mathbf{R}^n) \, ,$$

and this much exploited effect is known as *local smoothing*.

The same is true on many other noncompact manifolds under *non-trapping* assumptions. In fact, any trapping (e.g. presence of closed geodesics) will destroy local smoothing. Using recent results obtained by Stéphane Nonnenmacher and the speaker we show that local smoothing with $H^{1/2}$ replaced by $H^{1/2-\epsilon}$ can be obtained under the assumptions on the dimension of the trapped set, or more generally on the topological pressure of the classical flow. The connection with local smoothing comes through resolvent estimates and hence it is closely related to pseudospectral issues.