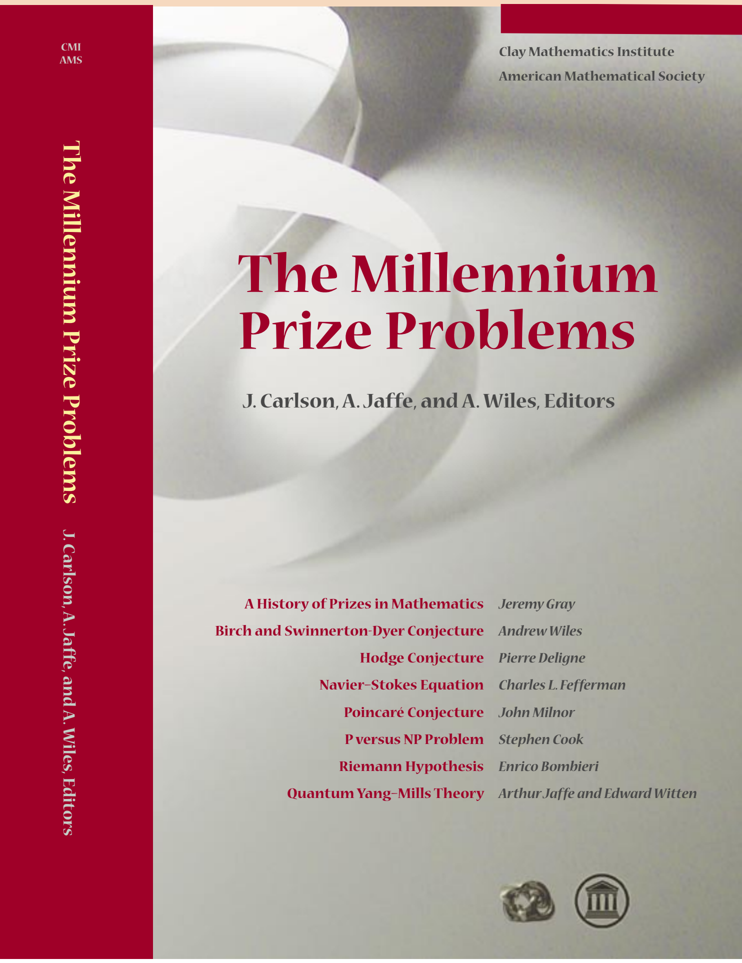
The expected publication date is July 23, 2006. Copies of the book will be available at the ICM in Madrid, August 22–30, 2006, and also sold at www.ams.org/bookstore

The Millennium Prize Problems



On August 8, 1900, at the second International Congress of Mathematicians in Paris, David Hilbert delivered the famous lecture in which he described twenty-three problems that were to play an influential role in future mathematical research. A century later, on May 24, 2000, at a meeting at the Collège de France, the Clay Mathematics Institute announced the creation of a US$7 million prize fund for the solution of seven important classic problems that have resisted solution. The prize fund is divided equally among the seven problems. There is no time limit for their solution.

The Millennium Prize Problems were selected by the founding Scientific Advisory Board of CMI – Alain Connes, Arthur Jaffe, Andrew Wiles, and Edward Witten – after consultation with other leading mathematicians. Their aim was somewhat different than that of Hilbert: not to define new challenges, but to record some of the most difficult issues with which mathematicians were struggling at the turn of the second millennium; to recognize achievement in mathematics of historical dimension; to elevate in the consciousness of the general public the fact that in mathematics the frontier is still open and abounds in important unsolved problems; and to emphasize the importance of working toward a solution of the deepest, most difficult problems.

The Millennium Prize Problems gives the official description of each of the seven problems and the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

The seven Millennium Prize Problems range from the oldest, the Riemann Hypothesis, a problem in number theory stated in 1859, to the youngest, the **P** versus **NP** problem, a problem in theoretical computer science stated in 1971. Informal descriptions are given below. See the individual articles in the book for official statements and background material. See also www.claymath.org/millennium

The Birch and Swinnerton-Dyer Conjecture: Let C be an elliptic curve over the rational numbers. The order to which its L function vanishes at $s = 1$ is the rank of the group of rational points.

The Hodge Conjecture: A rational cohomology class of type (p,p) on a projective algebraic manifold is represented by a rational sum of algebraic subvarieties.

The Navier–Stokes Equation. Show that the Navier–Stokes equations on Euclidean 3-space have a unique, smooth, finite energy solution for all time greater than or equal to zero, given smooth, divergence-free, initial conditions which “decay rapidly at large distances.” Alternatively, show that there is no such solution.

Poincaré Conjecture: a closed, compact, simply connected three-dimensional manifold is homeomorphic to the three-dimensional sphere.

P versus **NP** Problem: If a proposed solution to a problem can always be verified in polynomial time, can a solution always be found in polynomial time?

Riemann Hypothesis: Let s be a zero of the Riemann zeta function different from one of the trivial zeros $z = -2, -4, -6, \dots$. Then the real part of s is 1/2.

Quantum Yang–Mills Theory: Prove that for any compact simple group G, quantum Yang–Mills theory exists on 4-space and has a mass gap $\Delta > 0$.