Asymptotic Invariants of Groups
Oxford, 13-17 April 2015

Titles and Abstracts

Miklos Abert
A spectral Strong Approximation Theorem for measure preserving actions

Goulnara Arzhantseva
Expanders, small cancellation labellings, and infinite monster groups.

Nir Avni
Counting representations of arithmetic groups
I'll talk about the rate of growth of the number of d-dimensional representations of a higher-rank arithmetic group $G(\mathbb{Z})$, as d tends to infinity. This asymptotics is related to the distribution of random commutators in the congruence quotients of $G(\mathbb{Z})$, the singular values of random matrices in $\mathrm{Lie}(G)$, and the geometry of the character varieties of G.

Yiftach Barnea
Old and New Results on Subgroup Growth in Pro-p Groups.
I will survey our current knowledge about subgroup growth in pro-p growth. I will present some new solutions to long standing open problems in the area and I will talk about some important open questions. This is joint work (in progress) with Benjamin Klopsch and Jan-Christoph Schlage-Puchta.

Ian Biringer
Invariant measures on the space of all Riemannian manifolds.
We will discuss 'unimodular' measures on the space of all pointed Riemannian manifolds (M, p). These measures can be described in different ways: through a conservation of mass formula, via transverse measures on foliated spaces, or as measures that (when lifted to the space of unit tangent bundles of Riemannian manifolds) are invariant under geodesic flow. Unimodular measures are useful as they serve as limiting objects for the distribution of geometries seen near randomly chosen base points in a very large closed Riemannian manifold M. Applications to sequences of closed Riemannian manifolds (in particular, certain locally symmetric spaces) will be given.

Pierre-Emmanuel Caprace

Locally {linear compact} groups

Gabor Elek

Infinite dimensional modular representations of finite groups via Sylvester rank functions.

Abstract: Let k be a finite field and G be a finite group. If $\mathrm{char}(k)$ divides the order of the group, then (as opposed to the complex case) one might have infinitely many different indecomposable finite dimensional representations of G over k. The idea to compactify the space of indecomposable representations using Z/n-valued Sylvester rank functions goes back to Schofield (furthermore to Crawley-Boevey's work on endofinite modules). In our talk we show that for certain groups the compactification of the space of indecomposables via real valued Sylvester rank functions can be completely characterized by representations of the groups in the hyperfinite II_1 algebra over the field k (up to weak conjugacy). The proof suggests that there exist some strange connections between tame representation type and amenability resp. Sylvester rank functions and invariant random subgroups.

Misha Ershov

Tarski numbers of groups.

Tsachik Gelander

Counting hyperbolic manifolds (up to commensurability).

Gromov and Piatetski-Shapiro proved existence of finite volume non-arithmetic hyperbolic manifolds of any given dimension. In dimension four and higher, we show that there are about v^v such manifolds of volume at most v, considered up to commensurability. Since the number of arithmetic ones tends to be polynomial, almost all hyperbolic manifolds are non-arithmetic in an appropriate sense. Moreover, by restricting attention to non-compact manifolds, our result implies the same growth type for the number of quasi-isometry classes of lattices in $\mathrm{SO}(n, 1)$. Our method involves a geometric graph-of-spaces construction that relies on arithmetic properties of certain quadratic forms.

A joint work with Arie Levit.

Yair Glasner

Invariant random subgroups in linear groups.

Assume that G is a countable linear group with a simple, center free, Zariski closure. An IRS on G is a random subgroup whose law is invariant under conjugation by G. We will say that an IRS is non-trivial if it is non-trivial almost surely. I will discuss the following theorems:

Theorem A: The intersection of two, independent non-trivial IRS is also non-trivial.

Theorem B: There exists a non-discrete group topology on G of which every non-trivial IRS is supported on open subgroups.

The talk will focus on the case where G is a non abelian free group, where one can see many of the main ideas of the proof, but avoid some of the technical complications.

Andrei Jaikin-Zapirain

Approximation by subgroups of finite index and the Hanna Neumann conjecture.

Let F be a free group (pro-p group) and U and W two finitely generated subgroups (closed subgroups) of F. The Strengthened Hanna Neumann conjecture says that

$$\sum_{x \in U\backslash F/W} (rk(U \cap xWx^{-1}) - 1) \leq (rk(U) - 1)(rk(W) - 1).$$

This conjecture was proved in the case of abstract groups independently by J. Friedman and I. Mineyev in 2011.

In my talk I will explain the main ideas of the proof of the conjecture in pro-p context and also of a new proof in the abstract case.

Martin Kassabov

Intersection growth and zeta functions for nilpotent groups.

Intersection growth concerns the asymptotic behavior of the index $f(n)$ of the intersection of all (normal) subgroups of a group G that have index n. We obtain the growth type of this function in the case of high rank arithmetic groups. In the case of f.g torsion free, nilpotent groups G these numbers can be combined in a Dirichlet series and define a ζ function for the groups. Such zeta functions has many nice properties and are better behaved than the analogous function coming from subgroup growth or from representation growth.

Dessislava Kochloukova

Volume gradients and homology in towers of residually-free groups.

I will present the results from the joint work with Martin Bridson, arXiv:1309.1877. One of the main results is the calculation of the L^2-Betti numbers in dimension $\leq m-1$ of all residually free groups of type FP_m, using Luck's aproximation theorem. If the time permits I will discuss some new corollaries of the techniques introduced in the above preprint.

Alex Lubotzky

Isoperimetric inequalities for Ramanujan complexes and topological expanders

Expander graphs have been intensively studied in the last four decades. In recent years a high dimensional theory of expanders has emerged, and several generalizations have been studied. Among them stand out coboundary expansion and topological expansion. It is known that for every d there are unbounded degree simplicial complexes of dimension d with these properties. However, a major open problem, due to Gromov, is whether bounded degree high dimensional expanders exist for $d \geq 2$. We present an explicit construction of bounded degree complexes of dimension $d = 2$ which are topological expanders, thus answering Gromov's question for this case. This is obtained by proving several isoperimetric/systolic inequalities for 3-dimensional Ramanujan complexes. (Joint work with Tali Kaufman and David Kazhdan).

Wolfgang Lück

Universal torsion, L^2-invariants, polytopes and the Thurston norm

We introduce universal torsion which is defined for L^2-acyclic manifolds with torsionfree fundamental group and takes values in certain K_1-groups of a skew field associated to the integral group ring. It encompasses well-know invariants such as the Alexander polynomial and L^2-torsion. We discuss also twisted L^2-torsion and higher order Alexander polynomials which can also be derived from the universal invariant and assign certain polytopes to the universal torsion. This gives especially in dimension 3 interesting invariants which recover for instance the Thurston norm.

Denis Osin

Acylindrical hyperbolicity and groups with positive first L^2-Betti number.

The main purpose of my talk will be to discuss the following question: Is every finitely presented groups with positive first L^2-Betti number acylindrically hyperbolic? I will explain the rationale behind this question and present some partial results. In particular, I will show that the answer is positive for residually finite groups.

Jesse Peterson

Connes' character rigidity conjecture for lattices in higher rank groups.

We show that lattices in higher rank center-free simple Lie groups are operator algebraic superrigid, i.e., any unitary representation of the lattice which generates a II_1 factor extends to a representation of its group von Neumann algebra. This generalizes results of Margulis and Stuck-Zimmer, and answers in the affirmative a conjecture of Connes.

Mark Sapir

On the Jones' subgroup of R. Thompson group F.

This is a joint work with Gili Golan. Recently Vaughan Jones showed that the R. Thompson group F encodes in a natural way all knots and links in $\mathbb{R}^3$, and a certain subgroup $\overrightarrow{F}$ of F encodes all oriented knots and links. We answer several questions of Jones about $\overrightarrow{F}$. In particular we prove that the subgroup $\overrightarrow{F}$ is generated by x_0x_1, x_1x_2, x_2x_3 (where $x_i, i \in \mathbb{N}$ are the standard generators of F) and is isomorphic to F_3, the analog of F where all slopes are powers of 3 and break points are 3-adic rationals. We also show that $\overrightarrow{F}$ coincides with its commensurator. Hence the linearization of the permutational representation of F on $F/\overrightarrow{F}$ is irreducible. Finally we show how to replace 3 in the above results by an arbitrary n, and to construct a series of irreducible representations of F defined in a similar way.

Andreas Thom

Approximation of Betti numbers for totally disconnected groups.

Balint Virag

Dyson's spike and spectral measure of groups

Consider the graph of the integers with independent random edge weights. In 1953 Dyson showed that for exactly solvable cases even a small amount of randomness results in a logarithmic spike in the spectral measure.

With Marcin Kotowski, we prove that this phenomenon holds in great generality. As a result, we find that the Novikov-Schubin invariant is zero for lattices in the Lie group SOL.