D-modules, Geometric Representation Theory,
and Arithmetic Applications
4-8 December 2017
University of Oxford

Abstracts

Tomoyuki Abe (University of Tokyo)

Title: Arithmetic D-modules and its applications

Abstract: We will review the theory of arithmetic D-modules. The aim of this talk is to explain how to establish the theory for arbitrary separated scheme over a perfect field of positive characteristic. I will also discuss some unsolved questions in the theory.

Francesco Baldassarri (University of Padua)

Title: Closed categories of modules over generalized Huber rings

Abstract: see last page

Dan Ciubotaru (University of Oxford)

Title: Adams-Barbasch-Vogan classification

Abstract: In this expository talk, I aim to present some elements of the classification of irreducible characters of a real reductive group as in the work of Adams, Barbasch, and Vogan. The ABV picture replaces the space of usual Langlands parameters for irreducible characters of a real group with another geometric space better suited for Kazhdan-Lusztig geometry. The motivation of ABV is to provide a natural and precise formulation of the local part of Arthur's conjectures (and verify a large part of the conjectures) regarding the parameterization of the unitary representations which are of interest for global applications.

Richard Crew (University of Florida)

Title: Arithmetic D-modules on adic formal schemes.

Abstract: I will describe how Berthelot's theory of arithmetic D-modules can be extended to a certain class of morphisms of formally smooth adic schemes (the morphisms are not necessarily adic). I will then show how the construction of closed and open tubes extends to this setting. If time permits I will describe the construction of generalized dagger algebras and rigid cohomology.

Dmitry Gourevitch (Weizmann Institute)

Title: Harmonic analyses on spherical varieties – multiplicity bounds

Abstract: Let F be a local field of characteristic zero (Archimedean or not) and let G be a reductive algebraic F-group and H be an algebraic F-subgroup. Consider the space of smooth functions on G/H as a representation of G. We will discuss the multiplicities of this representation - when are they finite, when is there a universal bound on them and when this bound is one.

If time permits, we will also discuss the main components of some proofs. In the Archimedean case, D-modules are used in the proofs. Another central tool in the proof, for all local fields is the space of relative characters of H-invariant generalized vectors, that are certain generalized functions on G. In the Archimedean case, D-modules are also used in the analyses of this space.

Ian Grojnowski (University of Cambridge)

Title:

Abstract:

Christine Huyghe (University of Strasbourg)

Title: Coadmissible D-modules over Zariski-Riemann spaces

Abstract: In this talk we will describe some properties of the category of coadmissible modules over a Zariski-Riemann space. This is joint work with Matthias Strauch and Tobias Schmidt.

Bernard Le Stum (University of Rennes)

Title: An introduction to rigid cohomology

Abstract: This is a vey elementary lecture. Starting from a basic example, we will try to compute the number of points of a variety by hand. We will then introduce the Zeta function and show how it can help solve the problem. After that, we will explain how rigid cohomology can be used to compute this Zeta function. Only in the end, we will give the general definition of rigid cohomology.

Title: p-adic confluence and twisted operators

Abstract: This is a joint work with Adolfo Quirós. We recover Pulita's confluence theorem as a consequence of new D-module formalism. More precisely, we are able to define the notion of a twisted differential operator of given radius and show that this notion is essentially independent of the choice of the twist. By specializing the twist to a usual differential operator or to a quantum differential operator, we are able to derive an equivalence between differential equations and q-difference equations of the same radius. The construction is at the same time completely natural and very explicit.

Ruochuan Liu (Beijing ICMR)

Title: p-adic Riemann-Hilbert correspondence, de Rham comparison and periods on Shimura varieties

Abstract: In the previous work with Xinwen Zhu we construct a p-adic analogue of the classical Riemann-Hilbert correspondence. As a by-product the de Rham periods of a general Shimura variety are obtained. In a recent joint work with Hansheng Diao, Kai-Wen Lan and Xinwen Zhu, we further establish a logarithmic version of the correspondence which enables us to establish the de Rham comparison theorem with coefficients for quasi-projective varieties and compare the de Rham periods and complex periods for a general Shimura variety.

Adriano Marmora (University of Strasbourg)

Title: Product formula for p-adic epsilon factors.

Abstract: Let X be proper and smooth curve on a finite field of characteristic p and l be a prime different from p. In 1987, Laumon proved a formula, conjectured by Deligne, which correlates the constant appearing in the functional equation of the L-function of an l-adic sheaf over X, with the product of local data (the epsilon factors) at the points of X. In this talk, we report on the analogue of this formula in rigid cohomology, which I have proved in a joint work with Tomoyuki Abe. This formula has been used by T. Abe to prove the Langlands correspondence for isocrystals and the existence of crystalline companion on curves.

Dragan Milicic (University of Utah)

Title: D-modules and Harish-Chandra modules for real groups I, II

Abstract: There are two natural constructions of Harish-Chandra modules. One is homological - it is Zuckerman's cohomological induction. The other is algebro-geometric and is based on the localization theory of Beilinson and Bernstein. These two approaches are connected by the duality theorem of Hecht, Milicic, Schmid and Wolf. This allows a uniform algebro-geometric approach to the classification of Harish-Chandra modules and study of their properties.

Depending on available time, in the second talk, we are going to discuss some specific examples of this program.

Andrea Pulita (University of Grenoble)

Title: Index and Riemann Hurwitz formulas

Abstract: In a recent sequence of works we have provided necessary and sufficient conditions for the finite dimensionality of the de Rham cohomology of a certain large class of differential equations over quasi-smooth Berkovich curves. We also have obtained an Index formula of Grothendieck-Ogg-Shafarevich type. We prove how such a formula implies the Riemann-Hurwitz formula. In a second instance, we show how the Riemann-Hurwitz formula can be used in certain situations to improve our index theorem.

Peter Schneider (University of Münster)

Title: On the structure of derived Hecke algebras

Abstract: The smooth representation theory of a p-adic reductive group G with characteristic zero coefficients is very closely connected to the module theory of its (pro-p) Iwahori-Hecke algebra H = H(G). In the modular case, where the coefficients have characteristic p, this connection breaks down almost completely. In this talk I will first briefly describe a derived picture of the whole situation in which one recovers an equivalence between the module theory of a derived version H^\\bullet of H and the derived representation theory of G. Then I will survey joint work with R. Ollivier in which we study structural properties of the cohomology algebra of H^\\bullet.

Closed categories of modules over generalized Huber rings.

Francesco Baldassarri *

Oxford, Dec. 8, 2017

Abstract

This represents joint work with Maurizio Cailotto. We denote by $\mathcal{RR}$ the category of commutative rings separated and complete in a linear topology, and continuous ring-homomorphisms. We introduce a new condition, labelled c-op, on objects of $\mathcal{RR}$, more general than the condition of being adic or c-adic [2]. A *generalized* Huber ring differs from the standard notion ([3], [5]) in that it is only required to admit a ring of definition in $\mathcal{RR}^{(\text{c-op})}$. For an object k of $\mathcal{RR}^{(\text{c-op})}$ there exists a well-behaved full sub-category $\mathcal{LM}_k^{\text{can}}$ of the category $\mathcal{LM}_k^c$ of k-linearly topologized topological k-modules, separated and complete, consisting of objects, called *canonical*, whose topology is determined by k. For any k in $\mathcal{RR}^{(\text{c-op})}$, we single-out within the category $\mathcal{LM}_k^c$ the full subcategory $\mathcal{LM}_k^u$ (resp. $\mathcal{LM}_k^{\text{born}}$) of *uniform* (resp. of *bornological*) k-modules. We describe limits and colimits in these categories, and show that they are quasi-abelian and admit natural structures $\widehat{\otimes}_k^c$ (resp. $\widehat{\otimes}_k^u$, resp. $\widehat{\otimes}_k^b$) of symmetric monoidal categories. The simplest instance of our theory holds when $k = K^\circ$, for a non trivially valued non archimedean field K: we call this the *K-rigid case*. The main result of this paper is that, for any object k of $\mathcal{RR}^{(\text{c-op})}$, $\mathcal{LM}_k^{\text{born}}$ is a closed monoidal category. In the K-rigid case the full subcategory of topological K-vector spaces in $\mathcal{LM}_k^{\text{born}}$ reduces to the Bourbaki-style category $\mathcal{B}orn_K$ of bornological K-vector spaces [4], [1]. We generalize this to the case of an analytic (generalized) Huber ring R of pm-type with maximal ring of definition $R_0 = R^0$ and of topological R-modules in $\mathcal{LM}_{R^0}^{\text{born}}$ to obtain the category $\mathcal{B}orn_R$ of bornological R-modules. We show that $\mathcal{LM}_{R^0}^{\text{born}}$ (resp. $\mathcal{B}orn_R$) contains a natural generalization of weakly complete (resp. dagger) algebras over R_0 (resp. R).

References

BBB [1] Federico Bambozzi and Oren Ben-Bassat. Dagger geometry as Banach algebraic geometry J. Number Theory, 162, p. 391–462, 2016.

GR [2] Ofer Gabber and Lorenzo Ramero. Foundations for almost ring theory, ArXive 2016.

huber1 [3] Roland Huber A generalization of formal schemes and rigid analytic varieties. Mathematische Zeitschrift, 217 : 513–551, 1994.

schneider [4] Peter Schneider. Nonarchimedean functional analysis. *Springer Monographs in Mathematics*, Springer, 2002.

scholze [5] Peter Scholze. Perfectoid spaces. *Publ. Math. Inst. Hautes Études Sci.* 116: 245–313, 2012.

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