Clay Mathematics Institute

Functional Transcendence around Ax-Schanuel
September 29 - October 3, 2014

Abstracts of Talks

Daniel Bertrand (Institut de Mathematiques de Jussieu)
Title: Exponential and logarithmic Ax on semiabelian schemes

Abstract: I will report on joint work with A. Pillay, as well as with D. Masser and U. Zannier, giving a full description of these two extreme cases of Ax-Schanuel, for families of semiabelian surfaces. The functional transcendence is here controlled not only by the constant parts, but also by the constant images.

Alexandru Buium (University of New Mexico)
Title: Transcendental numbers as solutions to arithmetic differential equations

Abstract: Arithmetic differential equations are analogues of algebraic differential equations in which derivative operators acting on functions are replaced by Fermat quotient operators acting on numbers. Now various remarkable transcendental functions are solutions to algebraic differential equations; in this talk we will will show that, in a similar way, some remarkable transcendental numbers (including certain periods) are solutions to arithmetic differential equations. This opens up the possibility of understanding relations among periods via Galois groups of arithmetic differential equations.

Laura Capuano (Scuola Normale Superiore)
Title: On a Bombieri-Masser-Zannier theorem

Abstract: In this seminar we present an alternative proof of a theorem of Bombieri, Masser and Zannier of 1999 about intersecting a curve in the multiplicative group $\mathbb{G}_m^n$ with all the algebraic subgroups of of dimension n-2. To do that, we use a method introduced for the first time in 2008 by Pila and Zannier to give a new proof of Manin-Mumford conjecture and a theorem of Pila about counting rational points on certain transcendental varieties of Grassmannians. This method has been used afterwards to prove other cases of Unlikely Intersections problems in many different contexts.

Christopher Daw (IHES)
Title: Degrees of strongly special subvarieties and the Andre-Oort conjecture

Abstract: We give a new proof of the André–Oort conjecture under the generalised Riemann hypothesis. In fact, we generalise the strategy pioneered by Edixhoven, and implemented by Klingler and Yafaev, to

all special subvarieties. Thus, we remove ergodic theory from the proof of Klingler, Ullmo and Yafaev and replace it with tools from algebraic geometry. Our key ingredient is a lower bound for the degrees of strongly special subvarieties coming from Prasad's volume formula for S-arithmetic quotients of semis-imple groups.

Philipp Habegger (TU Darmstadt)
Title: Curves of genus 2 with bad reduction and complex multiplication
Joint work with F. Pazuki.

Jonathan Kirby (University of East Anglia)
Title: Local interdefinability and Ax-Schanuel

Abstract: We consider which holomorphic functions are locally definable from other holomorphic functions in the context of o-minimality. In the case of exponentiation and Weierstrass elliptic functions we are able to use Ax-Schanuel-type results to answer the question completely.

Bruno Klingler (Institut de Mathematiques de Jussieu)
Title: The hyperbolic Ax-Lindemann-Weierstrass conjecture
Title: Definability of the uniformizing map for Shimura varieties

Piotr Kowalski (Wroclaw University)
Title: Positive characteristic version of Ax's theorem

Abstract: Ax's theorem on the dimension of the intersection of an algebraic subvariety and a formal sub-group (Theorem 1F in "Some topics in differential algebraic geometry I...") implies Ax-Schanuel type transcendence results for a vast class of formal maps (including the exponential map on a semi-abelian variety). Ax stated and proved this theorem in the case of characteristic 0, but the statement is also meaningful for arbitrary characteristic and still implies Ax-Schanuel type transcendence results. I will discuss my work on characteristic version of Ax's theorem.

Angus Macintyre (Queen Mary University London)
Title: Comparing, and trying to distinguish, the complex and Zilber exponentials

Abstract. I will survey the attempts, all unsuccessful till now, to refute Zilber's bold conjecture connecting the classical complex exponential and his model-theoretically inspired exponential. These failures have led to interesting connections between Schanuel's Conjecture and conjectures in complex analysis.

David Masser (Mathematisches Institut Basel)
Title: Relative Manin-Mumford and integration in elementary terms

Abstract: Recently with Umberto Zannier we have shown that an algebraic curve in an abelian scheme with relative dimension at least two meets the torsion in at most finitely many points unless there is a good reason not to (and provided everything is defined over the algebraic numbers). This can be applied to Pell's Equation over polynomial rings and the integration of parametrized algebraic functions in elementary terms.

Ngaiming Mok (University Hong Kong)
Title: Asymptotics of Kähler metrics and hyperbolic Ax-Lindemann for arbitrary lattices

Martin Orr (University College London)
Title: Heights of pre-special points on Shimura varieties

Abstract: Two pieces are still needed to complete the Pila-Zannier approach to proving the André-Oort conjecture for general Shimura varieties: Galois bounds for special points (which are known assuming GRH) and a bound for height of the preimage of a special point in the fundamental domain. In work in progress with Chris Daw, I have been studying the second problem. In this talk I will outline our approach to the case of special points whose associated Hodge structure is irreducible.

Anand Pillay (University of Notre Dame)
Title: On functional Mordell-Lang in characteristic p

Abstract: I describe a new, essentially model-theoretic proof, of Mordell-Lang for function fields in characteristic p>0, which proceeds via a reduction to functional Manin-Mumford, and avoids recourse to results around Zariski geometries. Joint with F. Benoist and E. Bouscaren.

Harry Schmidt (University of Basel)
Title: Relative Manin-Mumford in additive extensions

Abstract: We will discuss the relative Manin-Mumford conjecture for families of two dimensional commutative algebraic groups. These will depend on one complex parameter λ and we are especially interested in the case of an additive extension of the Legendre family E_λ. We then have an exact sequence
$$0 \to \mathbb{G}_a \to G_\lambda \to E_\lambda \to 0$$
where $\mathbb{G}_a$ is the additive group $(\mathbb{C}, +)$. In this context the relative Manin-Mumford conjecture states that the intersection of a curve in G_λ with the set of torsion points is at most finite unless it is contained in a smaller family of algebraic subgroups in G_λ. It is possible to prove this by following the strategy employed by Masser and Zannier in their proof of the relative Manin-Mumford conjecture for the product of two Legendre families.

Michael Singer (North Carolina State University)
Title: Functional transcendence via groups and Galois theories

Abstract: In 1887, Hoelder proved that the Gamma function, defined by the difference equation $y(x+1) = x\,y(x)$, satisfies no nonzero polynomial differential equation with complex coefficients. In the last several years Galois theories have been developed that reprove this result and allow one to characterize when functions satisfying certain linear differential or difference equations also satisfy auxiliary difference or differential equations. These Galois theories allow one to reduce such kinds of questions to questions concerning linear differential or difference groups, that is groups of matrices whose entries are functions satisfying a fixed set of differential or difference equations. I will give an introduction to the theory of these groups and the related Galois theories and survey recent results applying these theories to questions of functional transcendence.

Sai Kee Yeung (Purdue University)
Title: Some arithmetic and geometric problems in complex ball quotients

Abstract: The purpose of the talk is to explain several instances in which arithmetics are used to understand geometry or vice versa, mainly on complex ball quotients. The first is the use of arithmetics in the classification of fake projective planes. The second is on finiteness of integral points on complex two ball quotients of finite volume. The last is to explain some examples for a problem of Lang on transcendence of the uniformizing radius of a hyperbolic Riemann surface.

Umberto Zannier (SNS Pisa)
Title: Unlikely intersections and Pellian polynomials

Abstract: We shall discuss a Pell equation $X^2-D(t)T^2=1$ in polynomials, and especially the distribution of complex polynomials $D(t)$ such that the equation has a solution in nonconstant polynomials $X(t)$, $Y(t)$. This topic is related to a conjecture of Pink on Unlikely Intersections.