Clay Mathematics Institute

Abstracts of Talks

Peter Allen (London School of Economics)
Title: Tight cycles and regular slices in dense hypergraphs

Abstract: We describe a general approach to the strong hypergraph regularity lemma, which we call 'regular slices', which avoids many of the usual technical complications and retains the features one would like to use in extremal hypergraph theory. This talk will avoid painful technical details in so far as that is possible and focus on an application, proving a hypergraph extension of the Erdős-Gallai theorem. This is joint work with Julia Böttcher, Oliver Cooley and Richard Mycroft.

Julia Böttcher (London School of Economics)
Title: An approximate version of the Tree Packing Conjecture

Abstract: We prove that for any pair of constants $\epsilon>0$ and Δ and for n sufficiently large, every family of trees of orders at most n, maximum degrees at most Δ, and with at most $n(n-1)/2$ edges in total, packs into $K_{1+\epsilon}n$. This implies asymptotic versions of the well-known tree packing conjecture of Gyárfás from 1976 and another tree packing conjecture of Ringel from 1963 for trees with bounded maximum degree. A novel random tree embedding process combined with the nibble method forms the core of the proof. In the talk I will in particular describe our random tree embedding process and explain how we use it to obtain our result. Joint work with Jan Hladky, Diana Piguet, Anusch Taraz.

David Ellis (Queen Mary)
Title: The structure of graphs which are locally lattice-like.

Abstract: There is a large body of results on random n-vertex, d-regular graphs, for d fixed and n large. Asymptotic formulae for the number of labelled and unlabelled d-regular graphs on n vertices were determined by Bender-Canfield and Bollobas, respectively. Amongst many other facts, it is now known that a random d-regular graph almost surely has trivial automorphism group, is almost surely Hamiltonian, is almost surely an expander and is almost surely d-connected. (These theorems were proved by Bollobas in the 1980's, the last also independently by Wormald.)

It is natural to ask what happens when we impose a local condition which is stronger than being regular. If G is a graph, and (F,w) is a rooted graph, we say that G is r-locally-F if for any vertex v of G, the subgraph of G induced by the ball of radius r around v is isomorphic to the subgraph of F induced by the ball of radius r around w. As an example, Benjamini and Georgakopoulos asked for a determination of the typical properties of an n-vertex graph which is r-locally-Z^2, for fixed r. We show that finite graphs which are d-locally-Z^d are covered normally by Z^d. We use this to give an explicit description of the finite graphs which are d-locally-Z^d. (Their global structure is very rigidly proscribed, in contrast to that of regular graphs.) Using methods and results from group theory and number theory, we obtain approximate formulae for the number of unlabelled n-vertex graphs which are d-locally-Z^d, and study some of their typical properties, e.g. the size of the largest component. Based on joint work with Itai Benjamini.

Dan Hefetz (University of Birmingham)
Title: Saturation games.

Abstract: A graph $G = (V,E)$ is said to be *saturated* with respect to a monotone increasing graph property $\mathcal{P}$, if $G \notin \mathcal{P}$ but $G \cup$ \{e\} $\in \mathcal{P}$ for every $e \in$\binom{V}{2}$\setminus E$. The *saturation game* $(n,\mathcal{P})$ is played as follows. Two players, called Mini and Max, progressively build a graph $G \subseteq K_n$, which does not satisfy $\mathcal{P}$. Starting with the empty graph on n vertices, the two players take turns adding edges e $\in$ \binom{$V(K_n)$}{2}$\setminus E(G)$, for which $G \cup$ \{e\} $\notin \mathcal{P}$, until no such edge exists (i.e. until G becomes $\mathcal{P}$-saturated), at which point the game is over. Max's goal is to maximize the length of the game, whereas Mini aims to minimize it. The *score* of the game, denoted by $s(n,\mathcal{P})$, is the number of edges in G at the end of the game, assuming both players follow their optimal strategies. We study the score of several natural saturation games. Based on joint work with Michael Krivelevich, Alon Naor and Miloš Stojaković.

Gil Kalai (Hebrew University of Jerusalem)
Title: Influence, thresholds and noise sensitivity

Abstract: We will consider the notions of influence and noise sensitivity of Boolean functions and discuss the connection with harmonic analysis, applications, and extensions. The influence of a variable (or a set of variables) on a function is the probability that changing the value of the variable(s) can change the value of the function. The noise-sensitivity of a function is the probability that for a random assignment to the variables adding a random independent noise will change the value of the function. We will look at some old and new results and open problems, and mention applications to sharp threshold phenomena, percolation, random graphs, voting, and computation: classic and quantum. The new results that I will present are based on joint works with Jeff Kahn, Jean Bourgain, and Elchanan Mossel, and on a work in progress with Guy Kindler.

Peter Keevash (University of Oxford)
Title: The existence of designs

Abstract: A Steiner Triple System on a set X is a collection T of 3-element subsets of X such that every pair of elements of X is contained in exactly one of the triples in T. An example considered by Plücker in 1835 is the affine plane of order three, which consists of 12 triples on a set of 9 points. Plücker observed that a necessary condition for the existence of a Steiner Triple System on a set with n elements is that n be congruent to 1 or 3 mod 6. In 1846, Kirkman showed that this necessary condition is also sufficient. In 1853, Steiner posed the natural generalisation of the question: given q and r, for which n is it possible

to choose a collection Q of q-element subsets of an n-element set X such that any r elements of X are contained in exactly one of the sets in Q? There are some natural necessary divisibility conditions generalising the necessary conditions for Steiner Triple Systems. The Existence Conjecture states that for all but finitely many n these divisibility conditions are also sufficient for the existence of general Steiner systems (and more generally designs). We prove the Existence Conjecture, and more generally, we show that the natural divisibility conditions are sufficient for clique decompositions of simplicial complexes that satisfy a certain pseudorandomness condition.

Daniel Král (University of Warwick)
Title: Structure of finitely forcible graphons

Abstract: Graphons, limits of convergent sequences of dense graphs, appear in various settings in extremal combinatorics, in particular in relation to the flag algebra method. In this talk, we investigate the structure of the topological space of typical vertices of finitely forcible graphons, i.e. those that are uniquely determined by finitely many subgraph densities. Lovasz and Szegedy (2011) conjectured that this space is always compact and has finite dimension. We construct a finitely forcible graphon with its space of typical vertices being neither compact nor finite dimensional. The talk is based on joint work with Roman Glebov, Tereza Klimosova and Jan Volec.

Daniela Kühn (University of Birmingham)
Title: Robustly expanding graphs and their applications

Abstract: Roughly speaking, a graph G is a robust expander if it is an expander (i.e. if the neighbourhood of every set of reasonable size is large), and if this expansion property is robust in the sense that one cannot destroy it by deleting a few vertices and edges. This notion (and its analogue for directed graphs) has turned out to be extremely fruitful in studying Hamilton cycles, Hamilton decompositions and more general subgraph embeddings. In my talk I will discuss this concept as well as some of the applications.

Choongbum Lee (MIT)
Title: Solution to the Erdős-Gyárfás conjecture on generalized Ramsey numbers

Abstract: Fix positive integers p and q with 2 ≤ q ≤ {p \choose 2}. An edge-coloring of the complete graph K_n is said to be a (p,q)-coloring if every K_p receives at least q different colors. The function $f(n,p,q)$ is the minimum number of colors that are needed for K_n to have a (p,q)-coloring. This function was introduced by Erdős and Shelah about 40 years ago, and Erdős and Gyárfás in 1997 initiated the systematic study of this function. In particular, they were interested how for each fixed p, the asymptotic behavior of $f(n,p,q)$ (as n goes to infinity) changes for different values of q. They proved several interesting results, but the question of determining the maximum q for which $f(n,p,q)$ is subpolynomial in n remained open. In this talk, we prove that the answer is q-1. Joint work with David Conlon, Jacob Fox, and Benny Sudakov.

Anita Liebenau (University of Warwick)
Title: What is Ramsey-equivalent to the clique?

Abstract: A graph G is Ramsey for H if every two-colouring of the edges of G contains a monochromatic copy of H. Two graphs H and H' are Ramsey-equivalent if every graph G is Ramsey for H if and only if it is Ramsey for H'. In the talk we discuss the problem of determining which graphs are Ramsey-equivalent to the complete graph K_k. In particular we prove that the only connected graph which is Ramsey-equivalent to K_k is itself. This gives a negative answer to a question of Szabó, Zumstein and Zürcher. For the proof we determine the smallest possible minimum degree that a minimal Ramsey graph for K_k . K_2 (the graph containing the clique with a hanging edge) can have. Joint work with Jacob Fox, Andrey Grinshpun, Yury Person and Tibor Szabó.

Nathan Linial (Hebrew University of Jerusalem)
Title: Local combinatorics

Po-Shen Loh (Carnegie Mellon University)
Title: Hamiltonian increasing paths in random edge orderings

Abstract: Let f be an ordering of the edges of K_n. An increasing path is a simple path (visiting each vertex at most once) such that the labels increase on successive edges. Let $S(f)$ be the number of edges in the longest increasing path. Chvátal and Komlós raised the question of estimating $m(n)$: the minimum value of $S(f)$ over all orderings f of K_n. The best known bounds on $m(n)$ are between about $\sqrt{n}$ and about $n/2$, due respectively to Graham and Kleitman, and to Calderbank, Chung, and Sturtevant. Although the problem is natural, it has seen essentially no progress for three decades.

In this paper, we consider the average case, when the ordering is chosen uniformly at random. We discover the surprising result that in the random setting, $S(f)$ often takes its maximum possible value of n-1 (visiting all of the vertices with a Hamiltonian increasing path). We prove that this occurs with probability at least about $1/e$. We also prove that with probability 1-o(1), there is an increasing path of length at least 0.85n, suggesting that this Hamiltonian (or near-Hamiltonian) phenomenon may hold asymptotically almost surely. Joint work with Misha Lavrov.

Eoin Long (University of Oxford)
Title: Frankl-Rödl type theorems for codes and permutations

Abstract: How large can a family $\mathcal{A} \subset \mathcal{P}[n]$ be if $|A \cap B| \neq t$ for all $A,B \in \mathcal{A}$? The Frankl-Rödl theorem shows that if $0 < \varepsilon n < t < (1/2-\varepsilon)n$ then $|\mathcal{A}| \leq (2-\delta)^n$, where $\delta = \delta(\varepsilon) > 0$. In this talk I will describe a new proof of this theorem. Our method extends to codes with forbidden distances, where over large alphabets our bound is significantly better than that obtained by Frankl and Rödl. One consequence of this result is a Frankl-Rödl type theorem for permutations with a forbidden distance. Joint work with Peter Keevash.

Deryk Osthus (University of Birmingham)
Title: Proof of two conjectures of Thomassen on tournaments

Abstract: We prove the following two conjectures of Thomassen on highly connected tournaments:

(i) For every k, there is an $f(k)$ so that every strongly $f(k)$-connected tournament contains k edge-disjoint Hamilton cycles (joint work with Kühn, Lapinskas and Patel).

(ii) For every k, there is an $f(k)$ so that every strongly $f(k)$-connected tournament has a vertex partition A,B for which both A and B induce a strongly k-connected tournament (joint with Kühn and Townsend).

Our proofs introduce the concept of `robust dominating structures', which will hopefully have further applications. I will also discuss related open problems on cycle factors and linkedness in tournaments.

Oleg Pikhurko (University of Warwick)
Title: Measurable equidecompositions via combinatorics and group theory

Abstract: Let $n>2$. We show that every two bounded subsets of R^n of the same measure and with non-emtpy interior can be equidecomposed using pieces that are measurable. Joint work with Lukasz Grabowski and Andras Mathe.

Oliver Riordan (University of Oxford)
Title: Counting connected hypergraphs via the probabilistic method

Abstract: For $n\geq r \geq 2$ and $0<p<1$, let $H^r_{n,p}$ be the random r-uniform hypergraph with vertex set $[n]$=\{1,2,. . .,n\} in which each of the {n\choose r} possible hyperedges is present independently with probability p, and normalize by writing $p=p(n)=\lambda\ (r-2)!n^{-r+1}$ where $\lambda=\lambda(n)$.

It is well known that $H^r_{n,p}$ undergoes a phase transition at $\lambda=1$ where a giant component first emerges, as shown by Erdős and Rényi in the graph case. For $\lambda>1$ constant, Behrisch, Coja-Oghlan and Kang not only found the asymptotic order of the giant component, but proved joint central and local limit theorems for its order (number of vertices) and size (number of edges). Their local limit result gives an asymptotic formula for the probability that the order and size take any pair of values in the `typical' range, and easily translates back to an asymptotic formula for the number of connected hypergraphs with a given number of vertices and a given `excess' or `nullity' (roughly speaking, number of extra `overlaps' compared to a tree) which is of the same order as the number of vertices.

As in the graph case, the situation when $\lambda\rightarrow 1$ from above is in many ways more delicate. In joint work with Bela Bollobas we have managed to extend the bivariate local limit result to the rest of the supercritical regime, i.e., when $\lambda(n)=1+\varepsilon(n)$ with $\varepsilon=o(1)$ and $\varepsilon^{3n}\rightarrow\infty$. This leads to an asymptotic formula for the number of connected hypergraphs with a given number n of vertices and given excess ℓ, whenever $\ell o(n)$ and $\ell\rightarrow\infty$. This formula generalizes one proved by Karonski and Luczak in 1997 by direct enumerative methods; their result applies only when ℓ=o(\log n/ \log\log n).

Wojciech Samotij (Tel Aviv University)
Title: The structure of a random metric space

Abstract: What does a typical metric space on n points look like? To formalize this question, we consider the set of all metric spaces on n points whose diameter is at most 2. Viewing every metric space as a vector of distances, this set becomes a convex polytope in $R^{\{n \text{ choose } 2\}}$, the so-called `metric polytope'. A random metric space is then a space chosen according to the normalized Lebesgue measure on this polytope. It is easy to see that the metric polytope contains the cube $[1,2]^{\{n \text{ choose } 2\}}$. Our main result is that it does not contain much more. Precisely, we show that a random metric space is very rigid, having all distances in an interval of the form [1-o(1), 2] with high probability. We consider several approaches to the problem, with varying degrees of accuracy, based on exchangeability, Szemeredi's regularity lemma, the Kovari-Sos-Turan theorem, and entropy techniques. Joint work with Gady Kozma, Tom Meyerovitch and Ron Peled.

Asaf Shapira (Tel Aviv University)
Title: A hierarchy of unavoidable tournaments

Yufei Zhao (MIT)
Title: Large deviations in random graphs: revisiting the infamous upper tail

Abstract: What is the probability that the number of triangles in the random graph $G(n,p)$ exceeds twice its mean? Janson and Ruciński (2002) called this problem the "infamous upper tail." Writing the probability as $\exp[-r(n,p)]$, already the order of the rate function $r(n,p)$ was a longstanding open problem when $p=o(1)$, finally settled in 2012 by Chatterjee and by DeMarco and Kahn.

We would like to understand the exact asymptotics of the rate function. The following variational problem can be related to this large deviation question: what is the minimum asymptotic p-relative entropy of a weighted graph on n vertices with triangle density at least $2p^3$? A beautiful large deviation framework of Chatterjee and Varadhan (2011) reduces upper tails for triangles to a limiting version of this problem for *fixed p*. A very recent breakthrough of Chatterjee and Dembo extended its validity to sparse random graphs, with $n^{-\alpha} \ll p \ll 1$ for an explicit $\alpha > 0$, and plausibly it holds for all $0 < \alpha < 1$.

We solve this variational problem asymptotically. As a corollary, this shows that the probability that $G(n,p)$, for $n^{-\alpha} \ll p \ll 1$, has twice as many triangles as its expectation is $\exp[-r(n,p)]$ where $r(n,p) \sim \frac{1}{3} n^2 p^2 \log(1/p)$. Joint work with Eyal Lubetzky.