New Frontiers in Probabilistic and Extremal Combinatorics

Abstracts of Talks

Matija Bucić (Princeton University)

Title: Equiangular lines in the exponential regime

Abstract: A family of lines passing through the origin in an inner product space is said to be *equiangular* if every pair of lines defines the same angle. In 1973, Lemmens and Seidel raised what has since become a central question in the study of equiangular lines in Euclidean spaces. They asked for the maximum number of equiangular lines in R^r with a common angle of arccos(1/(2k-1)) for any integer $k \geq 1$. We show that the answer equals r-1+floor((r-1)/(k-1)), provided that *r* is at least exponential in a polynomial in *k*. This improves upon a recent breakthrough of Jiang, Tidor, Yao, Zhang, and Zhao, who showed that this holds for *r* at least doubly exponential in a polynomial in *k*.

Joint work with Igor Balla.

David Conlon (California Institute of Technology)

Title: Around Sidorenko's conjecture

Abstract: We discuss recent progress on the study of a range of problems relating to homomorphism counts in graphs. Perhaps the most celebrated such problem is Sidorenko's conjecture, which says that the number of copies of any fixed bipartite graph in another graph of given density is asymptotically minimised by the random graph. As well as talking about some of the recent results on this conjecture, we will touch on the positive graph conjecture and the study of norming graphs. If time permits, we will also say a little about similar problems in an arithmetic context.

Tim Gowers (University of Cambridge)

Title: A proof of a conjecture of Marton

Abstract: Marton's conjecture is a sharp form of Freiman's theorem for F_p^n. It states that if A is a subset of F_p^n and |A+A| is at most C|A|, then there is a subgroup H of F_p^n of size at most |A| such that A can be covered by k cosets of H, where k depends polynomially on C. Recently, Ben Green, Freddie Manners, Terence Tao and I proved the conjecture using entropy methods. I shall describe the general strategy of the proof in the case p=2 and then explore as many of the details as time allows.

Ben Green (University of Oxford)

Title: On Sarkozy's theorem for shifted primes

Abstract: Suppose that N is large and that A is a subset of {1,..,N} which does not contain two elements x, y with x - y equal to p-1, p a prime. Then A has cardinality at most N^{1 - c}, for some absolute c > 0. I will discuss the history of this kind of question as well as some aspects of the proof of the stated result.

Annika Heckel (Uppsala University)

Title: Tame colourings of random graphs

Abstract: We discuss the chromatic number of the random graph G(n,1/2), and in particular `tame colourings' which we recently introduced to study counts of colourings. Our main result - a bound on the second moment of the number of such colourings - has several consequences: (1) It provides evidence for the `Zigzag conjecture' on the distribution of the chromatic number of G(n,1/2). (2) It can be used to improve the best lower concentration bound for the chromatic number of G(n,1/2), conjectured to be optimal. (3) For roughly 97% of all values of n, it can be used to solve a $100 question of Erdős on the cochromatic number.

Joint work with Konstantinos Panagiotou.

Oliver Janzer (University of Cambridge)

Title: Regular subgraphs at every density

Abstract: In 1975, Erdos and Sauer asked to estimate, for any constant *r*, the maximum number of edges that an *n*-vertex graph can have without containing an *r*-regular subgraph. Recently, Janzer and Sudakov proved that every *n*-vertex graph without an *r*-regular subgraph has at most $C_r n \log \log n$ edges, matching an earlier lower bound by Pyber, Rodl and Szemeredi and thereby resolving the Erdos-Sauer problem up to a constant depending on *r*. We prove that for every positive integer *r*, if *n* is sufficiently large, then every *n*-vertex graph without an *r*-regular subgraph has at most $Cr^2 n \log \log n$ edges. This bound is tight up to the value of *C* and hence it resolves the Erdos-Sauer problem up to an absolute constant.

We also obtain asymptotically tight results in the case where *r* is not necessarily a constant. We answer a question of Rodl and Wysocka from 1997 by determining asymptotically the smallest $d=d(r,n)$ such that every *n*-vertex graph with average degree at least *d* contains an *r*-regular subgraph. Perhaps surprisingly, the answer has a phase transition near $r=\log n$.

As a key step in the proof of the above results, we show that every *K*-almost-regular graph with average degree *d* contains an *r*-regular subgraph for some $r=\Omega_K(d)$.

Joint work with Debsoumya Chakraborti, Abhishek Methuku and Richard Montgomery.

Jeff Kahn (Rutgers University)

Title: Balancing extensions in posets of large width

Abstract: We revisit two old conjectures on linear extensions in finite partially ordered sets (posets) P. [A linear extension of P is a linear ordering compatible with the poset relations. A chain (antichain) is a totally ordered (unordered) set, and the width, w(P), is the maximum size of an antichain in P.]

Let $p(x \prec y)$ be the probability that x precedes y in a uniformly random linear extension, and set

$$\delta_{xy} = \min\{p(x \prec y), p(y \prec x)\} \text{ and } \delta(P) = \max \delta_{xy},$$

the max over distinct $x, y \in P$. The conjectures are:

Conjecture 1 (the “1/3-2/3 Conjecture”). If P *is not a chain then*

$$\delta(P) \geq 1/3.$$

Conjecture 2. *If* $w(P) \to \infty$, *then*

$$\delta(P) \to 1/2$$

(that is, $\delta(P) > 1/2 - o(1)$, where $o(1) \to 0$ as $w(P) \to \infty$).

We are still far from proving either of these, but make some interesting progress.

Joint with Max Aires.

Peter Keevash (University of Oxford)

Title: Isoperimetric stability and its applications

Abstract: We will survey some results on the stability of isoperimetric problems, broadly interpreted to include several directions in Combinatorics, Probability, Analysis, Geometry and Group Theory. The unifying theme of these stability results is describing the approximate structure implied by small growth. We will also discuss some applications to Extremal and Probabilistic Combinatorics and Group Theory.

These results are joint work with various combinations of Barber, Erde, van Hintum, Lifshitz, Long, Minzer, Roberts and Tiba.

Matthew Kwan (IST)

Title: Smoothed analysis for graph isomorphism

Abstract: From a theoretical point of view, graph isomorphism testing is a notoriously difficult problem, with no known polynomial-time algorithm. However, from a practical point of view, the problem is essentially solved: various elementary combinatorial “refinement” algorithms seem to be very efficient in practice. Some philosophical justification for this phenomenon is provided by a classical theorem of Babai, Erdős and Selkow, which shows that one of the simplest imaginable algorithms (underpinning all algorithms used in practice) is very effective "on average", in the sense that it can be used to distinguish a typical outcome of a random graph G(n,1/2) from any other graph.

We improve the Babai-Erdős-Selkow theorem in a few directions. In particular, in accordance with the *smoothed analysis* philosophy of Spielman and Teng, one of our main results is that for *any* graph G, simple combinatorial algorithms become effective after a tiny random perturbation to G (specifically, the addition and removal of about n random edges). The proof features a rather unusual combination of ideas, including a way to use the *k-core* of a random graph to enable percolation-type arguments.

Joint work with Michael Anastos and Benjamin Moore.

Imre Leader (University of Cambridge)

Title: Partial Shuffles by Transpositions

Abstract: Suppose we generate a random permutation by making a sequence of transpositions: we have a fixed sequence of transpositions and we apply each with a certain probability. How many transpositions do we need if we are to obtain a uniform random permutation? And what happens if we ask only that each element has the same probability of being in each position? We will discuss these and several related questions. (Joint work with Barnabas Janzer and Robert Johnson.)

Shoham Letzter (University College London)

Title: Packing subgraphs in dense regular graphs

Abstract: An H-packing in a graph G is a collection of pairwise vertex-disjoint copies of H in G. We prove that, for every bipartite graph H, every dense regular graph G has an H-packing covering all but a constant number of vertices in G. This resolves a problem of Kühn and Osthus from 2005.

This is joint work with Abhishek Methuku and Benny Sudakov.

Dor Minzer (Massachusetts Institute of Technology)

Title: The Lens of Abelian Embeddings.

Abstract: A predicate P:\Sigma^k \to {0,1} is said to be linearly embeddable if the set of assignments satisfying it can be embedded in an Abelian group. This talk will revolve around the notion of Abelian embeddings and different contexts it appears in. In particular, we will discuss:

1. Approximation algorithms and hardness of approximation for constraint satisfaction problems.
2. Multi-player parallel repetition theorems.

3. Additive combinatorics.

Mostly based on joint works with Amey Bhangale, Subhash Khot and Yang P. Liu.

Richard Montgomery (University of Warwick)

Approximate path decompositions of regular graphs

An edge decomposition of a graph G into copies of a graph F is a collection of edge-disjoint copies of F in G which cover every edge of G. In the last decade, significant progress has been made in determining simple properties of G which imply a edge decomposition into copies of different graphs F, but these results have mainly applied to dense host graphs G.

For sparse graphs G, it is natural to consider the case when F is a path. As any graph decomposable into paths of length d must have its number of edges divisible by d, we might ask: when d is odd, which d-regular graphs decompose into paths of length d? In 1957, Kotzig proved that any 3-regular graph G has a decomposition into paths of length 3 if and only if G has a perfect matching, but very limited progress has been made for odd d>3. This talk will discuss a recent approximate result, that almost all of the edges of a d-regular graph can be decomposed into paths of length roughly d.

This is joint work with Alp Müyesser, Alexey Pokrovskiy and Benny Sudakov.

Will Perkins (Georgia Institute of Technology)

Title: Triangle-free graphs in the critical regime

Abstract: A classic problem in probabilistic combinatorics is to estimate the probability that the random graph G(n,p) contains no triangles. This problem can be viewed as a question in large deviation theory or as a question in enumerative combinatorics (how many triangle-free graphs are there on n vertices with a specified edge density?). The asymptotics of the logarithm of this probability are known in two distinct regimes. When p>> 1/\sqrt{n}, at this level of accuracy the probability matches that of G(n,p) being bipartite; and when p<< 1/\sqrt{n}, Janson's Inequality gives the asymptotics of the log. I will discuss a new approach to estimating this probability in the "critical regime": when p = \Theta(1/\sqrt{n}). The approach uses ideas from statistical physics and algorithms and gives information about the typical structure of graphs drawn from the corresponding conditional distribution.

Based on joint work with Matthew Jenssen, Aditya Potukuchi, and Michael Simkin.

Alexey Pokrovskiy (University College London)

Title: Hyperstability in the Erdős-Sós Conjecture

Abstract: The Erdős-Sós Conjecture says that the Turan number of every k-edge tree is ≥ n(k-1)/2. For dense graphs, this conjecture is quite well understood, but for sparse graphs (when k=o(n)), some mystery remains. This talk will be about explaining the sparse case of this conjecture as well as its relatives in the special case of bounded degree trees.

Oliver Riordan (University of Oxford)

Title: Improved concentration of the chromatic number of certain random graphs

Abstract: A classical question in the theory of random graphs is `how much does the chromatic number of $G(n,1/2)$ vary?' For example, roughly what is its standard deviation σ_n? An old argument of Shamir and Spencer gives an upper bound of $O(\sqrt{n})$, improved by a logarithmic factor by Alon. For general *n*, a result with Annika Heckel implies that $n^{1/2}$ is tight up to log factors. However, according to the `zig-zag' conjecture σ_n is expected to vary between $n^{1/4+o(1)}$ and $n^{1/2+o(1)}$ as n varies. I will describe recent work with Rob Morris, building on work of Bollobás, Morris and Smith, giving an $O^*(n^{1/3})$ upper bound for certain values of *n*, the first bound beating $n^{1/2-o(1)}$, and almost matching the zig-zag conjecture for these *n*. The proof uses martingale methods, the entropy approach of Johansson, Kahn and Vu, the second moment method, and a new (we believe) way of thinking about the distribution of the independent sets in $G(n,1/2)$.

Lisa Sauermann (University of Bonn)

Title: Improving Behrend's construction for three-term progression free sets

Abstract: This talk discusses new lower bounds on the maximum size of subsets of{1,...,N} and of F_p^n not containing three-term arithmetic progressions. In the setting of {1,...,N}, our bounds give the first improvement upon a classical construction of Behrend from 1946 beyond lower-order factors. In the setting of F_p^n for a fixed prime p and large n, we obtain a lower bound of (cp)^n for some absolute constant c>1/2 (for c=1/2, such a bound can be obtained via classical constructions from the 1940s, but improving upon this has been a well-known open problem).
Joint work with Christian Elsholtz, Zach Hunter and Laura Proske.

Mehtaab Sawhney (Columbia University)

Title: Improved Bounds for Szemerédi Theorem

Abstract: We discuss recent improved bounds for Szemerédi's Theorem. The talk will seek to provide a gentle introduction to recent quantitative developments in higher order Fourier analysis. We provide a high level sketch for how the inverse theorem for the Gowers norm enters the picture and the starting points for the proof of the inverse theorem. Additionally, the talk (time permitting) will discuss how recent work of Leng on equidistribution of nilsequences enters the picture and is used. No background regarding nilsequences will be assumed.

Based on joint work with James Leng and Ashwin Sah.

Asaf Shapira (Tel Aviv University)

Title: Regularity for Hypergraphs of Bounded VC_2 Dimension

Abstract: While Szemeredi's graph regularity lemma is an indispensable tool for studying extremal problems in graph theory, using it comes with an hefty price, since in the worst case, a graph can have regular partitions of tower-type order. A remarkable result of Alon-Fischer-Newman and Lovasz-Szegedy states that for graphs of bounded VC dimension, one can reduce the tower-type bounds to polynomial.

The graph regularity lemma has been extended to the setting of k-uniform hypergraphs by Gowers, Nagle-Rodl-Schacht-Skokan and Tao. Unfortunately, these lemmas come with even larger Ackermann-type bounds. Fox-Pach-Suk considered a strong notion of hypergraph VC dimension and proved that hypergraphs with a bounded VC dimension of this type have regular partitions of polynomial size.

In the past decade, a weaker (and combinatorially more natural) notion of dimension, called VC_2 dimension, has been extensively studied. In particular, Terry and Wolf asked if one can improve the bounds for 3-graph regularity when the hypergraph has bounded VC_2 dimension. In this talk I will answer this question.

Joint work with Yuval Wigderson.

Jacques Verstraete (University of California, San Diego)

Title: Recent progress in Ramsey Theory

Abstract: The Ramsey number $r(s,t)$ denotes the minimum N such that in any red-blue coloring of the edges of the complete graph K_N, there exists a red K_s or a blue K_t. While the study of these quantities goes back almost one hundred years, to early papers of Ramsey and Erd\H{o}s and Szekeres, the long-standing conjecture of Erd\H{o}s that $r(s,t)$ has order of magnitude close to $t^{s - 1}$ as $t \

rightarrow \infty$ remains open in general. It took roughly sixty years before the order of magnitude of $r(3,t)$ was determined by Jeong Han Kim, who showed $r(3,t)$ has order of magnitude $t^2/(\log t)$ as $t \rightarrow \infty$. In this talk, we discuss a variety of new techniques, including mention of the proof that for some constants $a,b > 0$ and $t \geq 2$,

$$ a\frac{t^3}{(\log t)^4} \leq r(4,t) \leq b\frac{t^3}{(\log t)^2}, $$

as well as new progress on other Ramsey numbers, on Erd\H{o}s-Rogers functions, Ramsey minimal graphs, and on coloring hypergraphs.

Joint work in part with David Conlon, Sam Mattheus, and Dhruv Mubayi.