New Advances in the Langlands Program: Geometry and Arithmetic

Abstracts of Talks

George Boxer (Imperial College London)

Title: Modularity theorems for abelian surfaces

Abstract: We prove the modularity of a positive proportion of abelian surfaces over the rationals. A key ingredient is recent advances in p-adic Eichler-Shimura theory due to Pan, and further developed by Rodríguez Camargo. This is joint work with Calegari, Gee, and Pilloni.

Matthew Emerton (University of Chicago)

Title: Aspects of p-adic categorical local Langlands for GL_2(Q_p)

Abstract: The categorical p-adic local Langlands correspondence has been established for the group GL_2(Q_p) in joint work of the speaker with Andrea Dotto and Toby Gee. In this talk I will describe some aspects of the categorical correspondence, including its relationship to Taylor--Wiles--Kisin patching, to the work of Colmez and Paskunas, and to recent work of Johansson--Newton--Wang-Erickson. I also hope to illustrate what aspects of the GL_2(Q_p) situation are expected to carry over to the case of GL_2(Q_{p^f}), and what aspects are not.

Tony Feng (University of California, Berkeley)

Title: Modular functoriality in the Local Langlands correspondence

Abstract: I will talk about some results towards Langlands functoriality in the Fargues-Scholze correspondence between a group and its fixed subgroup under a prime order automorphism, in defining characteristic. In particular, using tools from Smith theory and modular representation theory, I establish some conjectures of Treumann-Venkatesh giving a generalization of the Jacquet functor to such situations. This also has applications to calculating the Fargues-Scholze parameters of unramified Yu types.

Jessica Fintzen (University of Bonn)

Title: Reducing the category of representations of p-adic groups to depth-zero

Abstract: An explicit understanding of the category of all (smooth, complex) representations of p-adic groups provides an important tool in the construction of an explicit and a categorical local Langlands correspondence and also has applications to the study of automorphic forms. The category of representations of p-adic groups decomposes into subcategories, called Bernstein blocks. I will give an overview of what we know about the structure of the Bernstein blocks. In particular, I will discuss a joint project with Adler, Mishra and Ohara in which we show that general Bernstein blocks are equivalent to much better understood depth-zero Bernstein blocks. This is achieved via an isomorphism of Hecke algebras and allows to reduce a lot of questions about the (category of) representations of p-adic groups to problems about representations of finite groups of Lie type, where answers are often already known or easier to achieve.

Toby Gee (Imperial College London)

Title: The Reduction modulo p of Crystalline Breuil—Kisin Modules

Abstract: I will discuss joint work with Mark Kisin on the mod p Breuil—Kisin modules arising from crystalline Galois representations, and the connection with the weight part of Serre's conjecture.

Ian Gleason (MPIM)

Title: Stacks of p-adic Shtukas and the Categorical Local Langlands

Abstract: In recent years, there have been two independent efforts to formulate categorical versions of the local Langlands correspondence. On one hand, Fargues and Scholze constructed a local Langlands category using diamonds and G-bundles on the Fargues–Fontaine curve; we call this the analytic category. On the other hand, Hemo and Zhu constructed a local Langlands category using perfect algebraic geometry and the Kottwitz stack parametrizing isocrystals with G-structure; we call this the schematic category.
Both the schematic and analytic categories share some key features: they are stratified by elements of the Kottwitz set, and the sheaf theory, stratum by stratum, agrees with the derived category of smooth representations of the associated p-adic group.

It is expected that the analytic and schematic categories are equivalent. In this talk, we explicate the geometry of the special fiber of the stack of p-adic shtukas. The upshot is that, with this stack, one can define a functor from the schematic category to the analytic category, which should provide the sought-after equivalence.

The ideas discussed in this talk arise from our collaboration with Ivanov and our ongoing collaboration with Hamann, Ivanov, Lourenço, and Zou.

Teruhisa Koshikawa (Kyoto University)

Title: A-parameters and eigensheaves

Abstract: In a joint project with Bertoloni Meli, we conjecture existsnce of some variant of Hecke eigensheaves for A-parameters in a generalized sense, extending Fargue's conjecture for discrete L-parametres. In fact, we construct candidate ind-coherent sheaves on the spectral side. After recalling the conjectrue, I will discuss the case of GL(2). Time permitting, I will mention a general construction and a related project on cuspidal coherent sheaves.

Si Ying Lee (Stanford University)

Title: Stacks of p-isogenies with G-structure

Abstract: I will talk about work in progress with Keerthi Madapusi to construct quasi-smooth stacks of p-isogenies with G-structure over Shimura varieties of abelian type at hyperspecial level. These stacks should be thought of as integral Hecke correspondences, and we describe how a geometric formalism associated with such stacks should allow us to realize various kinds of integral actions of the spherical Hecke algebra.

Jeffrey Manning (Imperial College London)

Title: Congruence modules in higher codimension

Abstract: In his work on modularity lifting, Wiles developed a numerical criterion for a map R->T of finite O-algebras, for O a DVR, to be an isomorphism of complete intersections. This criterion is in terms of a congruence module associated to a map T->O, and the cotangent space of the composition R->O.

In this talk I will present joint work with Srikanth Iyengar and Chandrashekhar Khare which generalizes the notion of congruence modules to maps A->O for O-algebras A of positive dimension over O, and proves a corresponding numerical isomorphism criterion, generalizing the classical numerical criterion. Applications include proving integral R=T theorems at non-minimal level for Bianchi forms or for weight one modular forms on Shimura curves. Time permitting, I will also discuss further work relating our congruence modules to special values of L-function and Galois cohomology, and a potential connection with the Bloch-Kato conjecture.

Alice Pozzi (University of Bristol)

Title: Modular generating series for Heegner objects

Abstract: Rigid meromorphic cocycles are cocycles for certain p-arithmetic groups acting on p-adic symmetric spaces. Their values at "special points" are conjectured to belong to class fields of some suitable global fields. These constructions mimic the formalism of special cycles for Shimura varieties that admit p-adic uniformisation. In this talk, we discuss certain modular generating series for values of rigid cocycles in a framework involving biquadratic extensions as well as a factorisation formula for their spectral decomposition in terms of L-values.

Juan Esteban Rodriguez Camargo (Columbia Univesity)

Title: Integral Hodge-Tate decompositions of p-adic Siegel modular forms

Abstract: There are two natural sources of families of p-adic modular forms. One arises from the coherent cohomology of Igusa varieties and the other one from the completed cohomology of Shimura varieties. In this talk I will explain how both constructions can be compared via the Hodge-Tate structure of completed cohomology thanks to the recent higher Hida theory of Boxer and Pilloni. Joint work in progress with Ana Caraiani and James Newton.

Peter Scholze (University of Bonn)

Title: The Habiro ring of a number field

Abstract: A question at the back of my mind has long been whether the q-deformation of de Rham cohomology is defined over the Habiro ring -- the completion of Z[q] at all roots of unity. In fact, this question was explicitly asked after my talk on q-de Rham cohomology at the Clay Research Conference several years ago. There is an easy definition for Artin motives, i.e. for number fields, but it did not seem to work in higher dimension. Completely unexpectedly, Garoufalidis and Zagier had found explicit elements of these Habiro rings H_F of number fields F, coming from perturbative Chern-Simons theory! I will report on joint work with them and Wheeler proving this and some more general results. Most mysteriously, this involves a certain kind of "exponentiated regulator" map $K_3(F) \to \mathrm{Pic}(H_F)$.

Christopher Skinner (Princeton University)

Title: Classes in the Galois cohomology of automorphic Galois representations

Abstract: Much of the progress on linking special L-values with orders or ranks of Selmer groups has come via new constructions of Galois cohomology classes satisfying particularly nice properties (eg., prescribed non-trivial ramification or the norm relations of Euler systems). In this talk I will describe some recent constructions of such classes in the context of Galois representations appearing in the cohomology of Shimura varieties.

Naomi Sweeting (Harvard University)

Title: A new bipartite Euler system and Selmer ranks for some four-dimensional symplectic Galois representations

Abstract: I will describe the construction of a new bipartite Euler system for GSp_4 and its inner forms, based on the special cycles appearing in the Kudla program (for instance, Shimura curves on Siegel threefolds). This leads to new results towards the Bloch-Kato Conjectures in ranks 0 and 1 for four-dimensional Galois representations associated to Siegel modular forms of parallel weights (3,3). The key ingredients of the proof are the structure of the supersingular locus for special fibers of GSpin_5 Shimura varieties at primes of both good and bad reduction, along with the representation theory of theta lifts underlying Kudla's program. To avoid imposing full-image hypotheses on the Galois representations, a new method of arithmetic level raising based on "relative deformation theory" is also developed.

Jack Thorne (University of Cambridge)

Title: The Ramanujan Conjecture for Bianchi Modular Forms

Abstract: The Ramanujan Conjecture for a Bianchi modular form over an imaginary quadratic field was proved in the lowest weight ("weight 2") case in the 10-author paper, by proving the potential automorphy of the symmetric powers of associated 2-dimensional compatible systems of Galois representations. Treating the higher weight situation presents difficulties due to the paucity of congruences between Bianchi modular forms of different weights. I will explain these difficulties and how they can be avoided to indeed prove the Ramanujan Conjecture in the general case. This is joint work with Boxer, Calegari, Gee, and Newton.

Pol van Hoften (VU Amsterdam)

Title: Igusa stacks and exotic Hecke correspondences

Abstract: Xiao and Zhu have conjectured the existence of exotic Hecke correspondences between the mod p fibers of different Shimura varieties. In this talk I will present a conjectural relationship between the Igusa stacks for different Shimura varieties, which implies the conjecture of Xiao—Zhu. I will then discuss a proof of this conjecture for a large class of Shimura varieties, and give applications to the global Jacquet—Langlands correspondence. This is joint work in progress with Jack Sempliner.

Andrew Wiles (University of Oxford)

Title: Non-abelian descent and modularity

Abstract: I will present a new approach to modularity based on the trace formula and using some ergodic and analytic arguments.

Sarah Zerbes (ETH Zürich)

Title: On the Bloch—Kato conjecture for Asai representations

Abstract: I will talk about joint work with David Loeffler and Giada Grossi, where we prove cases of the Bloch—Kato conjecture in analytic rank 0 for the Asai representation associated to quadratic Hilbert modular forms. I will also mention some applications to the Iwasawa Main Conjecture for the symmetric square of an elliptic curve.

Mingjia Zhang (Princeton University)

Title: Igusa stacks and Shimura varieties

Abstract: Recently, verifying a conjecture of Peter Scholze, geometric objects named Igusa stacks have been constructed as companions to Shimura varieties. They turn out to provide interesting information about both the geometry and cohomology of p-adic Shimura varieties. In this talk, I would like to survey some recent or ongoing work related to Igusa stacks.