

# The P=W Conjecture in Non Abelian Hodge Theory

## Abstracts of Talks

**Ben Davison** (University of Edinburgh)

Title: The stacky nonabelian Hodge correspondence

Abstract: The nonabelian hodge isomorphism provides a homeomorphism of underlying topological (analytic) spaces between the moduli space of semistable Higgs bundles on a smooth projective curve and the moduli space of representations of the fundamental group of the curve. In particular, whatever your favourite topological invariant is, it is the same for these two moduli spaces.

Passing to the stacks of semistable Higgs bundles and representations of the fundamental group, we no longer have such a homeomorphism. I will explain how it is possible nonetheless to show that these stacks have isomorphic Borel-Moore homology, by showing that they have isomorphic BPS cohomology, by relating this BPS cohomology to intersection cohomology of the coarse moduli spaces on both sides. I will explain how this produces a (still conjectural!) set of equivalences between various incarnations of the P=W conjecture.

**Camilla Felisetti** (University of Modena and Reggio Emilia)

Title: Intersection cohomology of moduli of rank 2 vector bundles

Abstract: Intersection cohomology is a topological notion adapted to the description of singular topological spaces, and the Decomposition Theorem for algebraic maps is a key tool in the subject. Motivated by the work of Mozgovoy and Reineke, in joint work with Andras Szenes and Olga Trapeznikova, we give a complete description of the intersection cohomology of the moduli space of vector bundles of rank 2 via a detailed analysis of the Decomposition Theorem applied to a certain map from parabolic bundles. We also give a new formula for the intersection Betti numbers of these moduli spaces, which has a clear geometric meaning.

**Andres Fernandez Herrero** (University of Pennsylvania)

Title: Decomposition theorem for the logarithmic Hitchin fibration

Abstract: The focus of this talk will be the moduli space of semistable logarithmic principal G-Higgs bundles on a smooth curve. For any given degree  $d$  in the algebraic fundamental group of  $G$ , we will explain a uniform description of the decomposition theorem for the corresponding Hitchin fibration of degree  $d$  logarithmic G-Higgs bundles. This is based on joint work with Mark Andrea de Cataldo, Roberto Fringueli and Mirko Mauri.

**Andrew Harder** (Lehigh University)

Perversity, weight, and mirror symmetry

Mirror symmetry predicts that if  $X$  and  $Y$  are a mirror pair of compact Calabi—Yau varieties, then there is an equality between their Hodge numbers after a rotation of the Hodge diamond. If  $X$  and  $Y$  are log Calabi—Yau varieties one expects that the weight filtration on the cohomology of  $X$  appears as the perverse Leray filtration of the affinization map on  $Y$ . I'll describe a collection of conjectures in this direction. Recent results with S. Lee explain how to recast Hodge number mirror symmetry as a true isomorphism on the level of tropical cohomology groups. Considering tropical versions of the perverse Leray and weight filtrations the relationship between perversity and weight in mirror symmetry can be stated precisely. I will state this conjecture and describe work in progress towards proving it.

**Victoria Hoskins** (Radboud University Nijmegen)

Title: Higgs moduli spaces with abelian motives and applications

In the recent proofs of the  $P = W$  conjecture, the tautological generation of the cohomology of the Higgs moduli space plays a crucial role. I will explain a motivic analogue of this tautological generation: that the motive of the  $GL$ -Higgs moduli space is generated by the motive of the curve and the motive of the  $SL$ -Higgs moduli space is generated by the motives of certain étale covers of the curve. In particular, these Higgs moduli spaces have abelian motives. I will then explain various applications using conservativity results for abelian motives, such as how to obtain motivic formulas in low rank and how to prove motivic chi-independence and mirror symmetry. This is joint work with Simon Pepin Lehalleur and partially also with Lie Fu.

**Tasuki Kinjo** (Kyoto University)

Title: Cohomological Hall induction for  $(-1)$ -shifted symplectic stacks

Abstract: In this talk, I will explain a construction of the cohomological Hall induction for the critical cohomology of  $(-1)$ -shifted symplectic stacks, which recovers the cohomological Hall algebra multiplication for the cohomological Donaldson-Thomas invariants of 3-Calabi-Yau categories as a special case. This talk is based on my joint work with Hyeonjun Park and Pavel Safronov.

I will use the backboard for the talk, and I am happy to talk any day of the week.

**Emmanuel Letellier** (Jussieu)

Title:  $PGL_n$ -character varieties and Langlands duality over finite fields

Abstract: We expect that philosophically the weight filtration on the cohomology of  $G$ -character varieties interpolates the structure coefficients of two rings: the center of the group algebra of  $G(q)$  and the character ring of  $G^*(q)$ . In this talk we will make this precise in the case of  $G = PGL_n$ . This is a joint work with Tommaso Scognamiglio.

**Anton Mellit** (University of Vienna)

Title: Cohomology rings of character varieties

Abstract: I will give an introduction and present some recent progress towards understanding the cohomology rings of character varieties of Riemann surfaces, such as the proof of the  $P=W$  conjecture and the computation of the zero-dimensional COHA. In the case of punctured sphere I will present an explicit description relating the cohomology rings to the Hilbert scheme of  $C^2$ , refining conjectures of Hausel-Letellier-Rodriguez-Villegas and Chuang-Diaconescu-Donagi-Pantev.

**Tony Pantev** (University of Pennsylvania)

Title: Classifying abstract Higgs bundles

Abstract: I will report on an ongoing joint work with D. Arinkin and R. Donagi which seeks to develop a systematic approach for studying principal Higgs bundles in the abstract coefficient-free setting. I will discuss the associated Hitchin maps and the comparison with the traditional Higgs moduli. This provides a universal framework for understanding how compactifications, Fourier-Mukai dualities, and Chern filtrations for Hitchin fibrations are affected by the change of the coefficients of the Higgs bundles.

**Fernando Rodriguez Villegas** (ICTP)

Title: Tensor product multiplicities for  $GL_n$  and  $U_n$  over finite fields

**Abstract:** In this talk we will discuss the multiplicities of the tensor product of irreducible characters of the general linear and unitary groups over a finite field  $K$ . These multiplicities are given by polynomials in  $q$ , the size of  $K$ . For generic unipotent characters these polynomials are related to Poincaré polynomial of certain quiver varieties. We will describe how we can go from the generic to the general case uncovering a very interesting underlying combinatorics. Particularly interesting is the case of the tensor square of the Steinberg representation. The combinatorics relates to the symmetric group action on the standard permutohedron and yields a symmetric function analogue of the Tutte polynomial of the complete graph. This is joint work with Emmanuel Letellier.

**Olivier Schiffmann** (University Paris-Saclay)

**Title:** Cohomological Hall algebras of Higgs bundles on curves, BPS algebras and cuspidal functions

**Abstract:** We will present a conjecture relating the space of cuspidal functions for the groups  $GL_n$  over the function field of a smooth projective curve, as a module over the Hecke algebra, and the space of simple root vectors of the BPS Lie algebra associated to a complex curve (of the same genus), as a module over the cohomological Hall algebra of zero-dimensional sheaves.

**Junliang Shen** (Yale University)

**Title:** Tautological classes, perverse filtration, and Fourier transform

**Abstract:** Tautological classes have played a crucial role in the study of cohomology of the moduli of stable vector bundles on Riemann surfaces since the work of Mumford and Atiyah-Bott over 40 years ago. In recent years, the study of cohomological aspects of non-abelian Hodge theory (the  $P=W$  conjecture) and enumerative geometry (Gopakumar-Vafa theory) links tautological classes to a very different structure --- the perverse filtration governed by the topology of an abelian fibration. I will start by reviewing this connection. Then, I will discuss a Fourier transform theory, developed in collaboration with Davesh Maulik and Qizheng Yin, that explains this relationship. If time allows, further open questions will be discussed.

**Olga Trapeznikova** (IST)

**Title:** Decomposition of the parabolic map to the moduli of semistable bundles: the topology of the fibers and the intersection forms.

**Abstract:** The study of the intersection cohomology of GIT quotients goes back to the works of Frances Kirwan in the 80's. In joint work with Camilla Felisetti and Andras Szenes, we calculate the intersection cohomology of moduli spaces of semistable bundles on curves using the Decomposition Theorem applied to a certain map: the parabolic projection. In this talk, I will explain how a detailed analysis of the topology of this map leads to our calculation of the local systems appearing in the Decomposition Theorem.

**Qizheng Yin** (Peking University)

**Title:** A Beauville decomposition for the Hitchin system

**Abstract:** Beauville (and later Deninger-Murre) found a natural decomposition of the Chow ring/motive of an abelian scheme via Fourier transforms. I will present an extension of this theory to the abelian fibration given by the Hitchin system. Such extensions to abelian fibrations are typically met with two difficulties: 1) extending the Poincaré line bundle; 2) the problem of the “smaller supports”. I will explain how to circumvent these issues in this particular case. Joint work with Davesh Maulik and Junliang Shen.

**Siqing Zhang** (Institute for Advanced Study)

**Title:** What happens in positive characteristic?

**Abstract:** For a curve  $C$  in positive characteristic, the main players in the Non Abelian Hodge (NAH) Theory are the de Rham moduli of connections on  $C$ , the Dolbeault moduli of Higgs bundles on the Frobenius twist

$C'$  of  $C$ , the Picard stack of torsors under the regular centralizers on  $C'$ , and a torsor under the Picard stack. The stacky NAH isomorphism, which originated from the Geometric Langlands Program, is between the de Rham moduli and the twist of the Dolbeault moduli by the Picard stack-torsor. It does not preserve semistability. With Mark de Cataldo and Michael Groechenig and with Andres Fernandez Herrero, we can modify the stacky NAH isomorphism and obtain an isomorphism that preserves semistability. As a consequence, we show that the degree  $d$  Dolbeault moduli space and the degree  $pd$  de Rham moduli space have the same cohomology rings and the same intersection cohomology groups. With Mark de Cataldo, we study what happens if we add log poles. The picture is different: an Artin-Schreier type cover of the Hitchin base occurs.

**Zili Zhang** (Tongi Univesity)

Title: The  $P=W$  phenomenon for cluster varieties

Abstract: The cohomology of even-dimensional full rank cluster varieties of Louise type satisfy the curious hard Lefschetz property, which behave very similar to the character varieties. Such a resemblance potentially leads to a  $P=W$  type identity. In this talk, we will study the cohomology cluster varieties in a more general setting and establish a conjectural  $P=W$  type identity. I will provide various explicit cluster varieties and combinatorial-geometric constructions as evidence toward the  $P=W$  conjecture for cluster varieties.